



TECHNICAL NOTE

D-791

NUMERICAL SOLUTIONS FOR SUPERSONIC FLOW OF AN IDEAL
GAS AROUND BLUNT TWO-DIMENSIONAL BODIES

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SUMMARY

The method described is an inverse one; the shock shape is chosen and the solution proceeds downstream to a body. Bodies blunter than circular cylinders are readily accessible, and any adiabatic index can be chosen. The lower limit to the free-stream Mach number available in any case is determined by the extent of the subsonic field, which in turn depends upon the body shape. Some discussion of the stability of the numerical processes is given. A set of solutions for flows about circular cylinders at several Mach numbers and several values of the adiabatic index is included.

INTRODUCTION

A simple and fast method for calculation of the flow field between a blunt body at zero incidence to a supersonic stream of ideal gas and its attendant detached shock wave was presented in reference 1. Further work, including a catalog of solutions for the case of bodies with axial symmetry, was reported in reference 2. These solutions were primarily applicable to the subsonic part of the flow field extending from the line of symmetry, and ending in a region where the flow was supersonic. The sonic line had no particular significance in this work.

The problem of supersonic flow around a blunt nose has been one of considerable interest in aerodynamics, both from the standpoint of analytic approximation techniques (see refs. 3 and 4) and from that of numerical solutions of the full inviscid equations of motion. A survey of the then existing literature can be found in reference 1; reference 4 should also be seen. Current interest in this problem includes the desire to determine a solution for a complete configuration. This entails a continuation of the results found for the subsonic and transonic regions at the nose of the body around the shoulder and on into the purely supersonic flow downstream. Several methods for accomplishing this have been proposed in the literature (e.g., refs. 5, 6, 7), and some of these even include consideration of equilibrium and nonequilibrium real-gas effects. The importance of the results of such calculations is so great, however, that it was thought worthwhile to develop still another technique

for the solution. The present work is therefore a step toward the eventual calculation of flow about a whole configuration. In it, certain devices whose desirability was indicated by examination of the results of references 1 and 2 are used. Like the method of reference 1, it is indirect in that the calculation begins at a shock wave and proceeds inward until a body shape is determined. If this body is not of the required shape, a different shock is used, and the process repeated until the desired body shape is attained.

One difficulty that arose in the work of reference 1 had to do with the particular coordinate system chosen there. Specifically, this trouble arose when an attempt was made to calculate the flow behind a shock wave whose section was an oblate ellipse (shock for which $B_s > 1$; see ref. 1). Thus, it was decided to abandon the otherwise convenient set of orthogonal coordinates devised by Van Dyke (ref. 1) and to use an oblique set in which such a difficulty could not arise.

Another point of interest is in connection with the inherent instability to random error that is present in attempts to determine numerically the solution to a partial differential equation of elliptic type starting from initial, or Cauchy type, data. This is an ill-set problem of the type discussed by Hadamard in reference 8. An example of the effect of this instability is given in reference 1. Therefore, one must compromise between truncation error (or the error introduced when a difference analog is used for differentiation) and round-off error by using as large a mesh size as feasible. This results in some sparsity of data in the upper transonic region where a characteristics solution could be started. A smoothing operation was introduced into the calculation in an effort to get around the instability difficulty.

Finally, an attempt was made to increase the capability of the method to produce desired body shapes by incorporating a second parameter into the specification of shock-wave shape.

Only two-dimensional results are discussed herein, but the machine program is set up so that axially symmetric results are available by changing one number in the input data. Some checks with the results of reference 2 were made and appeared satisfactory. It should be noted that the present results carry the solution only slightly past the sonic line, and do not continue into the supersonic region downstream.

SYMBOLS

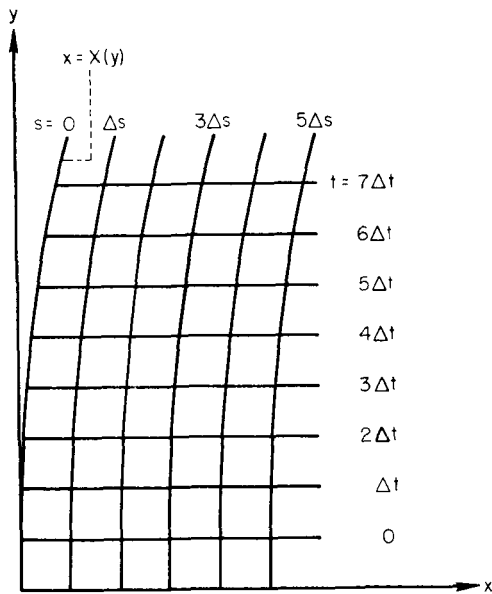
- A parameter affecting shock-wave shape (eq. (2a))
- B_s, B_b bluntness of shock or body, respectively (see eq. (2b))

c_p, c_v	constant specific heats at constant pressure and volume, respectively
$f(\psi)$	$\frac{p}{\rho^\gamma}$, (eq. (5b))
M, M_l	free stream and local Mach numbers, respectively
p	pressure $\frac{p}{\rho_\infty U_\infty^2}$
p_b	value of p on body surface
p_{st}	value of p at stagnation point
R_s, R_b	radius of curvature at nose of shock and body, respectively
S	entropy
s, t	oblique curvilinear coordinates (eq. (1))
u, v	x and y components of velocity, referred to free-stream speed U_∞
U_∞	velocity of free stream, along x axis
x, y	Cartesian coordinates, referred to R_s
$X(y)$	functional representation of curve which represents a shock wave (eq. (2a))
γ	ratio of specific heats, $\frac{c_p}{c_v}$
δ	stream angle, $\delta = \tan^{-1} \frac{v}{u}$
Δ	shock standoff distance
θ, θ_b	inclination of tangent to shock or body, respectively (see sketch (c))
ρ	density, referred to free-stream density ρ_∞
ψ	stream function, referred to $\rho_\infty U_\infty$
ω	$\frac{\psi}{\bar{y}}$

COORDINATE SYSTEM

To avoid some nonanalytic behavior of the solution which was associated with the orthogonal curvilinear coordinate system of reference 1, a different curvilinear coordinate system is used here. The requirement of orthogonality will be dropped. A typical example of the coordinate system is shown in sketch (a).

Here the curve $x = X(y)$ represents a possible shock-wave shape, and the new coordinates are simply measured off as shown from this original curve and from the axis of symmetry. Thus, set



Sketch (a)

$$\left. \begin{aligned} s &= x - X(y) & x &= s + X(t) \\ t &= y & y &= t \end{aligned} \right\} \quad (1)$$

where x, y are Cartesian coordinates made dimensionless by division by the radius of curvature R_s of the shock wave at its apex. Lines of constant s and t are shown in sketch (a). The (s, t) system is not orthogonal but the Jacobian of the transformation is unity so that there is nonanalyticity introduced by the coordinate transformation (1).

The curve $x = X(y)$ which represents a shock wave should have a derivative at each point, and its tangent should be vertical where it intersects the symmetry or stream axis. This latter requirement is made because only flow fields with

vertical symmetry are to be considered herein. Many possibilities are therefore available, but because of their simplicity, conic sections are chosen as the basic shapes. (Another attractive possibility, however, is the power law shapes.) The form chosen for $X(y)$ is now

$$x = X(y) = (1 + Ay^2) \frac{1 - \sqrt{1 - B_S y^2}}{B_S} \quad (2a)$$

Here, B_S is the bluntness parameter introduced in reference 1, and A is a second shape parameter which may be used to gain more flexibility in the choice of shock-wave shape. The type of conic used is determined by the bluntness B_S :

$$\left. \begin{array}{ll} B_S < 0 & \text{hyperbola} \\ B_S = 0 & \text{parabola} \\ 1 > B_S > 0 & \text{prolate ellipse} \\ B_S = 1 & \text{circle} \\ B_S > 1 & \text{oblate ellipse} \end{array} \right\} \quad (2b)$$

In cases where $B_S > 0$, it is just the square of the axis ratio of the ellipse.

An idea of the effect of the parameter A upon the shape of the curves chosen to represent the shock waves can be gained from sketch (b). The basic curve is a prolate ellipse with $B_S = 0.5$ and the other curves show the effect of positive and negative values of A .

EQUATIONS OF MOTION

The equations governing the steady flow of an ideal, nonviscous, nonheat-conducting gas in two dimensions can be written (subscripts denote differentiation)

Continuity:

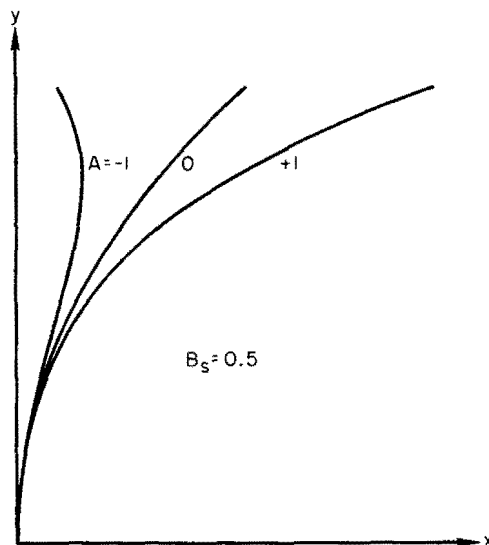
$$(\rho u)_x + (\rho v)_y = 0 \quad (3a)$$

Momentum:

$$\left. \begin{array}{l} uu_x + vu_y + \rho^{-1} p_x = 0 \\ uv_x + vv_y + \rho^{-1} p_y = 0 \end{array} \right\} \quad (3b)$$

Energy:

$$uS_x + vS_y = 0 \quad (3c)$$



Sketch (b)

Initial conditions for starting the numerical solution of equations (3) are obtained from the oblique-shock relations. First, however, equations (3) will be cast in a form appropriate to the coordinate system discussed in the previous section.

The continuity equation (3a) can be eliminated by the introduction of a (dimensionless) stream function $\psi(x,y)$ such that

$$\rho u = \psi_y ; \quad \rho v = -\psi_x \quad (4)$$

Since the energy equation (3c) implies that the entropy S is constant along streamlines, it follows that S is a function of ψ alone;

$$S = c_v \log \frac{p}{\rho^\gamma} = c_v \log f(\psi) \quad (5a)$$

The relation

$$\frac{p}{\rho^\gamma} = f(\psi) \quad (5b)$$

can be used to eliminate pressure p from the momentum equations (3b). The results of all this, and of replacing velocity components by the corresponding derivatives of the stream function, are as follows:

$$\psi_y \psi_{xy} - \psi_x \psi_{yy} - \psi_y^2 \frac{\rho_x}{\rho} + \psi_x \psi_y \frac{\rho_y}{\rho} + (\rho^{\gamma+1} f) \left[\gamma \frac{\rho_x}{\rho} + \psi_x \left(\frac{f'}{f} \right) \right] = 0 \quad (6a)$$

$$-\psi_y \psi_{xx} + \psi_x \psi_{xy} + \psi_x \psi_y \frac{\rho_x}{\rho} - \psi_x^2 \frac{\rho_y}{\rho} + (\rho^{\gamma+1} f) \left[\gamma \frac{\rho_y}{\rho} + \psi_y \left(\frac{f'}{f} \right) \right] = 0 \quad (6b)$$

It might be emphasized at this point that though there appear to be more unknowns than equations, in reality the entropy function f is a function of the stream function ψ . This follows from the energy relation, equation (3c). The principle of constant entropy along streamlines holds for equilibrium flows of a real gas as well as of a perfect gas, so that equations (6) apply to this situation also. The determination of the functional form of $f(\psi)$ will be considered later in the section on boundary conditions. However, it can be said that an analytic form for $f(\psi)$ can be found only for the perfect-gas case; for a real gas, numerical tables or functional approximations thereto must suffice.

The density derivatives can be eliminated from equations (6) to give a single second-order equation in the stream function ψ . To do this, Crocco's relation between vorticity and entropy gradient can be used, and the result is

$$\left(1 - \frac{u^2}{c^2} \right) \psi_{xx} - \frac{2uv}{c^2} \psi_{xy} + \left(1 - \frac{v^2}{c^2} \right) \psi_{yy} + \frac{c^2 \rho^2}{\gamma(\gamma-1)} \frac{f'}{f} = 0$$

where c is the local sound speed divided by U_∞ . There did not seem to be any advantage gained by working with this single equation rather than with the pair (6a) and (6b).

In the solution, the body contour will be located as the locus of zeros of the stream function. Unfortunately, ψ vanishes all along the symmetry axis since this is the streamline containing the stagnation point on the body. To avoid this, it is convenient to introduce a modified stream function $\omega(x,y)$, defined by

$$\omega = \frac{\psi}{y} \quad (7)$$

which vanishes only on the body. This new function has also a generally smaller variation than does ψ in the region of interest.

In terms of ω , equations (6) become

$$\begin{aligned} (\omega + y\omega_y) \left[(\omega_x + y\omega_{yx}) - (\omega + y\omega_y) \frac{\rho_x}{\rho} + y\omega_x \frac{\rho_y}{\rho} \right] - 2y\omega_y\omega_x \\ - y^2\omega_x\omega_{yy} + (\rho^{\gamma+1}f) \left[\gamma \frac{\rho_x}{\rho} + y\omega_x \left(\frac{f'}{f} \right) \right] = 0 \quad (8a) \end{aligned}$$

$$\begin{aligned} (\omega + y\omega_y) \left(-y\omega_{xx} + y\omega_x \frac{\rho_x}{\rho} \right) + y\omega_x(\omega_x + y\omega_{xy}) - y^2\omega_x^2 \frac{\rho_y}{\rho} \\ + (\rho^{\gamma+1}f) \left[\gamma \frac{\rho_y}{\rho} + (\omega + y\omega_y) \left(\frac{f'}{f} \right) \right] = 0 \quad (8b) \end{aligned}$$

It remains to transform equations (8) according to the variable change of equations (1). The results of this are the equations

$$\begin{aligned} tX'(\omega + t\omega_t)\omega_{ss} + \left\{ (\omega + t\omega_t)[\omega + t(\omega_t - X'\omega_s)] - \gamma(\rho^{\gamma+1}f) \right\} \frac{\rho_s}{\rho} \\ = t\omega_{ts}(\omega + t\omega_t + tX'\omega_s) - t^2\omega_s(\omega_{tt} - X'\omega_s) + \omega_s[\omega - t(\omega_t - X'\omega_s)] \\ + t\omega_s[\omega + t(\omega_t - X'\omega_s)] \frac{\rho_t}{\rho} + (\rho^{\gamma+1}f)t\omega_s \left(\frac{f'}{f} \right) \quad (9a) \end{aligned}$$

and

$$\begin{aligned} t(\omega + t\omega_t)\omega_{ss} - [t\omega_s(\omega + t\omega_t) - \gamma X'(\rho^{\gamma+1}f)] \frac{\rho_s}{\rho} \\ = t\omega_s \left(\omega_s + t\omega_{ts} - t\omega_s \frac{\rho_t}{\rho} \right) + (\rho^{\gamma+1}f) \left\{ \gamma \frac{\rho_t}{\rho} + [\omega + t(\omega_t - X'\omega_s)] \left(\frac{f'}{f} \right) \right\} \quad (9b) \end{aligned}$$

These are the equations whose numerical solution is sought. Before the method of solution is considered, the initial conditions will be discussed.

INITIAL CONDITIONS

If a step-by-step, or marching, method of numerical solution is used in the present problem, there are the possible alternatives of starting either at the body or at the shock. The former, or direct, method has much to recommend it since the body shape is nearly always prescribed. However, this is not enough information with which to start a solution process, so it is necessary to assume, for example, a pressure distribution on the body surface. The integration then proceeds until a shock wave can be inserted leading to a uniform free stream. If this cannot be done, the pressure distribution is modified and the process repeated until a shock wave can be successfully inserted. On the other hand, by starting from the shock, all the necessary initial data are known immediately and the integration can proceed until the body is reached. The iterations in this case are then made with altered shock shapes to get a desired body. Of the two, this indirect method seems somewhat the simpler to execute and is the one to be used here. Furthermore, if consideration is to be given to equilibrium, and, especially, to nonequilibrium flows of a real gas, then the development of the flow field from shock wave to body seems the more natural one to consider. Of course, the free-boundary aspect of this problem could be avoided if the work were done in terms of, say, ψ/y , or ω , and y as independent variables.

A
3
7
2

The initial conditions on the shock wave $s = 0$ will now be determined. Sketch (c) shows the shock wave and the local shock angle θ , determined from

$$\tan \theta = \frac{dy}{dx} = \frac{1}{X'} \quad (10)$$

or

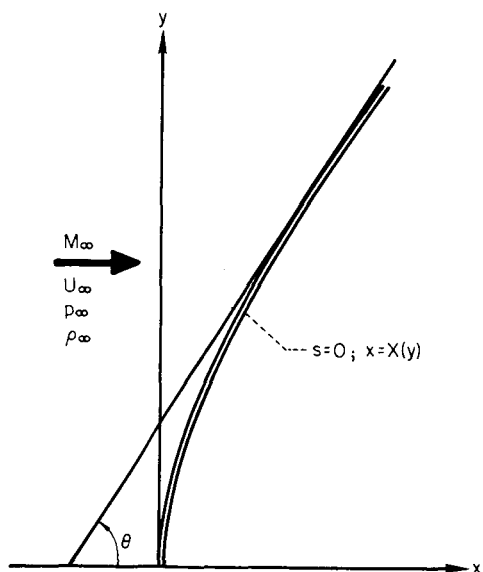
$$\sin^2 \theta = \frac{1}{1 + X'^2}, \quad \cos^2 \theta = \frac{X'^2}{1 + X'^2}$$

The value of the stream function ψ in the free stream ahead of the shock is $\psi = y$, hence $\omega = 1$ in the free stream. Since the stream function is continuous at the shock, this is also the starting value of ω just behind the shock;

$$\omega(0, t) = \omega^{(0)} = 1 \quad (11a)$$

From this, it follows that

$$\omega_t(0, t) = \omega_t^{(0)} = 0 \quad (11b)$$



Sketch (c)

In order to find the starting value of ω_s , use

$$\omega_s = \frac{\psi_x}{y} = - \frac{\rho v}{y}$$

so that the starting values of ρ, v need to be known. From reference 9,

$$v^{(0)} = 2 \cot \theta \frac{M^2 \sin^2 \theta - 1}{(\gamma + 1) M^2}; \quad \rho^{(0)} = \frac{(\gamma + 1) M^2 \sin^2 \theta}{2 + (\gamma - 1) M^2 \sin^2 \theta}$$

It follows that

$$\omega_s(0, t) = \omega_s^{(0)} = - \frac{2}{t} \cot \theta \frac{\sin^2 \theta (M^2 \sin^2 \theta - 1)}{2 + (\gamma - 1) M^2 \sin^2 \theta} \quad (11c)$$

Derivatives along the shock are found with the relation

$$\left. \frac{\partial}{\partial t} \right]_{\text{shock}} = \frac{d}{d\theta} \frac{d\theta}{dt} = - \frac{X'(t)}{1 + X'^2(t)} \frac{d}{d\theta}$$

The initial values of the quantities ω_{tt} , ω_{st} , and ρ_t/ρ can all be found with this formula; they are

$$\omega_{tt}^{(0)} = 0 \quad (11d)$$

$$\begin{aligned} \omega_{st}^{(0)} = & \frac{\sin 2\theta}{t^2} \frac{M^2 \sin^2 \theta - 1}{2 + (\gamma - 1) M^2 \sin^2 \theta} + \frac{1}{t} \frac{X''}{1 + X'^2} \left\{ 2 \cos 2\theta \frac{M^2 \sin^2 \theta - 1}{2 + (\gamma - 1) M^2 \sin^2 \theta} \right. \\ & \left. + \frac{(\gamma + 1) M^2 \sin^2 2\theta}{[2 + (\gamma - 1) M^2 \sin^2 \theta]^2} \right\} \end{aligned} \quad (11e)$$

$$\left(\frac{\rho_t}{\rho} \right)^{(0)} = - \frac{X''}{1 + X'^2} \left[2 \cot \theta - \frac{(\gamma - 1) M^2 \sin 2\theta}{2 + (\gamma - 1) M^2 \sin^2 \theta} \right] \quad (11f)$$

The only remaining quantity whose initial value needs to be known is the entropy function $f(\psi)$. The functional form of f must also be found. At the shock, one finds from, for example, reference 9 that

$$f^{(0)}(\psi) = \left(\frac{p}{\rho^\gamma}\right)^{(0)} = \frac{2\gamma M^2 \sin^2 \theta - (\gamma - 1)}{\gamma(\gamma + 1)M^2} \left[\frac{2 + (\gamma - 1)M^2 \sin^2 \theta}{(\gamma + 1)M^2 \sin^2 \theta} \right]^\gamma \quad (12a)$$

$$= \frac{2\gamma M^2 - (\gamma - 1)(1 + X'^2)}{\gamma(\gamma + 1)M^2(1 + X'^2)} \left[\frac{(\gamma - 1)M^2 + 2(1 + X'^2)}{(\gamma + 1)M^2} \right]^\gamma \quad (12b)$$

Now as to the functional form of f one reasons as follows. At the shock, f is known as a function of y (or t) through the slope function $X'(t)$ as in equation (12b). Also, the stream function ψ , hence ω , is known as a function of t along the shock. Hence, along this line, the functional form of $f(\psi)$ is known, and this functional form should persist throughout the flow field. This now allows the calculation of the initial values of the derivative f'/f . Since

$$\frac{d}{d\psi} = \frac{dX'^2}{d\psi} \frac{d}{dX'^2}$$

by logarithmic differentiation one has

$$\begin{aligned} \left(\frac{f'}{f}\right)^{(0)} &= \frac{dX'^2(\psi)}{d\psi} \left[-\frac{1}{1 + X'^2} - \frac{\gamma - 1}{2\gamma M^2 - (\gamma - 1)(1 + X'^2)} \right. \\ &\quad \left. + \frac{2}{2(1 + X'^2) + (\gamma - 1)M^2} \right] \end{aligned} \quad (12c)$$

The derivative $dX'^2/d\psi$ can be evaluated when the shape of the shock wave is prescribed. For the family of curves of equation (2a)

$$X'(y) = y \left(\frac{2AX(y)}{1 + Ay^2} + \frac{1 + Ay^2}{\sqrt{1 - B_S y^2}} \right) \quad (13a)$$

$$X''(y) = \frac{2AX(y)}{1 + Ay^2} + \frac{1 + 5Ay^2}{\sqrt{1 - B_S y^2}} + B_S y^2 \frac{1 + Ay^2}{(1 - B_S y^2)^{3/2}} \quad (13b)$$

There is now available enough information to start the numerical solution of equations (9).

NUMERICAL PROCEDURE

The differential equations to be integrated (eqs. (9)) are seen to be in the form

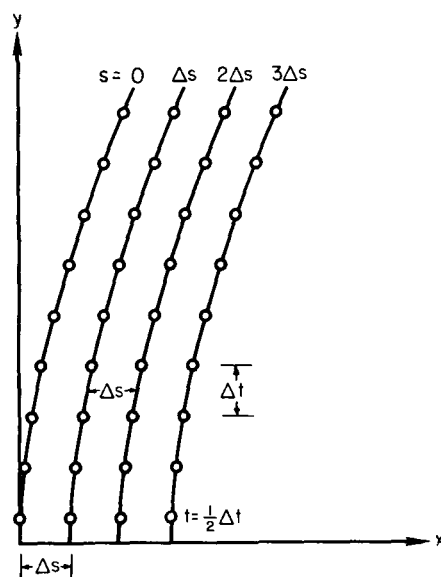
$$A_1 \omega_{SS} + B_1 \frac{\rho_S}{\rho} = C_1 \quad (14a)$$

$$A_2 \omega_{SS} + B_2 \frac{\rho_S}{\rho} = C_2 \quad (14b)$$

where the coefficients A_i, B_i, C_i involve only quantities whose values are known at the shock wave, or initial line, $s = 0$. Then, considering equations (14) as linear algebraic equations for ω_{ss} and ρ_s/ρ , these quantities can readily be determined at $s = 0$. There is then sufficient information to write truncated Taylor expansions for ω and ρ in terms of the independent variable s , the coefficients of which are known as functions of t . These expansions are used to project ω and ρ to the next coordinate line $s = \Delta s$, and the set of values of ω and ρ can then be differentiated numerically with respect to y . The whole process can be repeated now, expanding from $s = \Delta s$ to $s = 2\Delta s$, and so forth. Sketch (d) shows the arrangement of points used in the computations.

Notice that, to avoid difficulty at $t = 0$, the grid is displaced $\Delta t/2$ off the symmetry axis. Results can later be extrapolated to $y = 0$, making use of the vertical symmetry of the flow field. The present computing procedure is set up to allow for a maximum of 50 points to be chosen along the initial line $s = 0$. The interval Δt and number of points (≤ 50) chosen to start the process is arbitrary, except for the following considerations. The conic section, or other analytic curve, which is chosen to represent a shock wave will, in general, fail to represent it well at some distance from the symmetry axis. Thus, if the assumed shock is taken to be a circular arc, then only so much of the circle as has a slope $\tan \theta$ (see sketch (c)) which is $> (M^2 - 1)^{-1/2}$ can be used; that is,

the curve representing a shock wave must be inclined to the symmetry axis at an angle greater than the free-stream Mach angle, otherwise the shock relations used to calculate initial conditions do not apply. This situation is automatically adjusted in the machine program; the machine will drop sufficient points to ensure that the condition above is met.

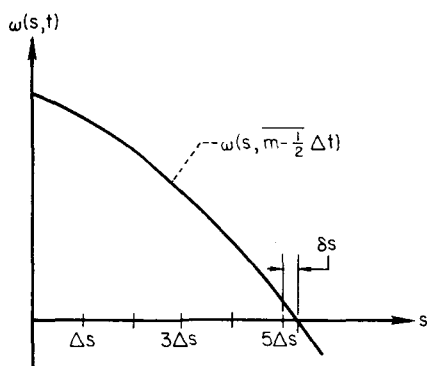


Sketch (d)

The above procedure can be repeated until at some step, say the $(n + 1)$, one or more negative values of ω appear along the coordinate line $s = (n + 1)\Delta s$. Since $\omega = \psi/y$, it is clear that the zero stream-line for certain values of t (or y) lies between the lines $s = n\Delta s$ and $s = (n + 1)\Delta s$. The location of this zero is then determined from the known data along $s = n\Delta s$ by solving for δs from the equation expressing that the truncated series for the stream function vanishes:

$$\omega(s + \delta s, t) = 0 = \omega(n\Delta s, t) + \overline{\delta s} \omega_s(n\Delta s, t) + \frac{1}{2} \overline{\delta s}^2 \omega_{ss}(n\Delta s, t)$$

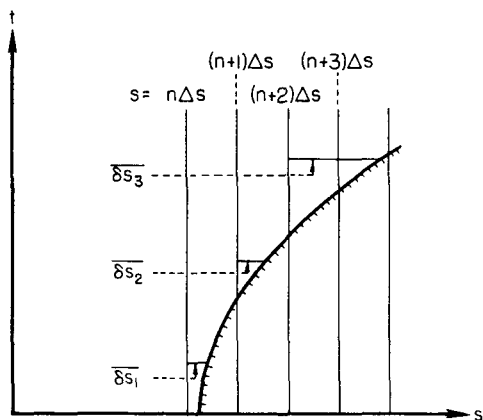
At each value of t , the body is now located at $s = n\Delta s + \delta s$, and the (s, t) coordinates are easily put in terms of the (x, y) system by means



Sketch (e)

determination now continues until the machine is stopped by some form of misbehavior of the solution. The usual form of misbehavior is found in the excessive growth of the second derivatives of ω , which occurs because of the penetration of the body surface after the first body calculation. Since there must be some sort of sourcelike singularity inside the zero streamline, calculation within this region will soon fail because of excessive growth of the stream function and especially its derivatives.

Sketch (f) illustrates some of the quantities involved in the body determination process.



Sketch (f)

In solving the linear equations (14) for ω_{ss} and ρ_s/ρ , it can happen that

$$A_1 B_2 - A_2 B_1 = 0$$

in which case the solution will fail. To explain this, one can derive the characteristic form for equations (14) and show that the above condition indicates that at the point in question, the coordinate line $s = \text{constant}$ is tangent to a characteristic line and the failure of the extrapolation procedure is justified.

This can therefore only occur in regions of supersonic flow, and can generally be avoided by a slight change in the interval size Δs or Δt .

Calculation of the Flow Variables

The density ρ is calculated at every point of the mesh. As in reference 1, an external iteration is performed on density to improve its

accuracy. Since the entropy function is also known at each mesh point, calculation of the pressure is immediate, as

$$p = \rho^\gamma f(\psi)$$

When the pressure at each point is known, the local Mach number is found by means of the formula (see ref. 9)

$$\begin{aligned} M_l^2 &= \frac{2}{\gamma - 1} \left(\frac{\rho}{p} \frac{p_{st}}{\rho_{st}} - 1 \right) \\ &= \frac{2}{\gamma - 1} \left[\frac{2 + (\gamma - 1)M^2}{2\gamma M^2} \frac{\rho}{p} - 1 \right] \end{aligned} \quad (15)$$

For cases in which the adiabatic index γ is taken as 1, formula (15) is modified to read

$$M_l^2 = M^2 \left(1 - \sin^2 \theta + \frac{1}{M^4 \sin^2 \theta} \right) + 2 \log \frac{\sin^2 \theta}{p} \quad (16)$$

where $\sin^2 \theta$ is the same function of ψ as in the calculation of $f(\psi)$.

The velocity components in the (x,y) system, u and v, as well as the stream angle $\delta = \tan^{-1}(v/u)$ are printed out in the machine calculation. These would be useful for plotting streamlines, as well as for adjoining a characteristics solution to the present one.

It was mentioned previously that flow variables are not calculated on the symmetry axis, but start $\Delta t/2$ off it. To extrapolate these values to the axis, the first two calculated points and the symmetry in $y = 0$ are used to determine a cubic parabola. The formula resulting is

$$A(s,0) = \frac{9}{8} A\left(s, \frac{1}{2} \Delta t\right) - \frac{1}{8} A\left(s, \frac{3}{2} \Delta t\right) \quad (17)$$

where $A(s,t)$ represents the physical quantity in question.

Characterization of Body Shape

The question of how to characterize a set of body points resulting from a calculation naturally arises. One way would be simply to plot these points and compare the results with a plot of the desired shape. This is a necessary procedure if the desired shape is not represented by

a simple formula. However, many nose shapes of interest are conic sections; in particular the circle is of considerable importance as a standard solution for comparison purposes. Thus, it is possible to gain some idea about the closeness of the calculated body shape to the desired one by a numerical process. The process used here is the same as that of reference 2; if the body is a conic, its equation is

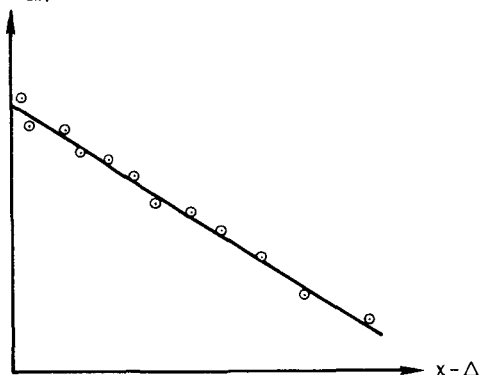
$$y^2 = 2 \frac{R_b}{R_s} (x - \Delta) - B_b(x - \Delta) \quad (18a)$$

where R_b is nose radius, B_b the bluntness, and Δ the standoff distance of the body. The quantity

$$\frac{y^2}{2(x - \Delta)} = \frac{R_b}{R_s} - \frac{B_b}{2} (x - \Delta) \quad (18b)$$

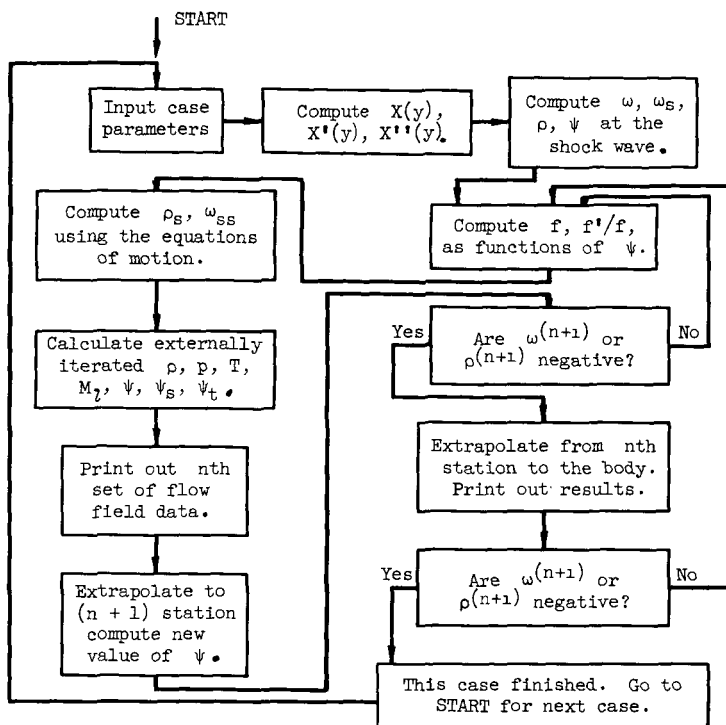
can be plotted versus $(x - \Delta)$, and the points approximated to by a straight line (see sketch (g)). The vertical intercept of the line is R_b/R_s , and its slope is $-(1/2)B_b$.

$\frac{y^2}{2(x - \Delta)}$



Sketch (g)

The machine program was set up to include this calculation by a least-squares fit, but this is not completely satisfactory because of occasional wildness of points, particularly near $x - \Delta = 0$. Thus, one can use the machine-calculated values of $y^2/2(x - \Delta)$ to make one's own fit. In addition, the program calculates y at each body value of x based upon the least-square values of B_b and R_b . These can be checked with the actual values and the closeness of the conic approximation evaluated.



Sketch (h)

Sketch (h) shows a generalized flow chart for the calculating procedure outlined above.

INSTABILITY AND SMOOTHING

An attempt was made to examine the effect of the inherent instability with respect to random error which is known to exist when a partial differential equation of elliptic type is solved numerically with boundary conditions of the Cauchy type. There is of course always a random error in rounding off to any number of figures which may be kept in a calculation, but in order to get a clearer notion of this effect, the starting values of ω (only) were deliberately perturbed from their correct values, and a solution was determined from these new initial conditions.

The case taken for this purpose was that which resulted in a circular cylinder at $M = 4$, $\gamma = 1.4$. The shock bluntness $B_s = -0.06$. Several runs were made with errors of different magnitude introduced, and each error was run at two interval sizes in the s direction. In the cases with the larger interval size, $\Delta s = 0.032$, six steps were required between the shock and the body. This number of steps was considered about optimum by Van Dyke in references 1 and 2. The smaller interval size, $\Delta s = 0.016$, required 13 steps from shock to body.

Values of the modified stream function $\omega(s,t)$ at $t = (1/2)\Delta t$ for the cases with $\omega[0,(1/2)\Delta t] = 1.0000000$ and $\omega[0,(1/2)\Delta t] = 1.0004883$ are shown in figure 1. It is seen that the artificial error has a considerable effect on the variation of ω , causing a change in standoff distance of about 13 percent when the interval size is $\Delta s = 0.032$. The differences in the case where $\Delta s = 0.016$ are much greater, and the solution was, in fact, halted by the error accumulation at $s = 0.160$. The difference between the solutions with no artificial error but with $\Delta s = 0.032$ and 0.016 are definitely noticeable; this is presumably due to the growth of error caused by rounding to eight figures, as well as to the different truncation errors. The standoff distance differs here by something less than 2 percent. Also shown in figure 1(a) is the variation of $\omega(s,0.11)$. At this vertical distance from the induced error, its effect was not visible for as much of the solution as was obtained ($\Delta s = 0.016$).

Figure 1(b) shows the variation of ω with t along $s = \text{constant}$ lines for the case $\Delta s = 0.016$. Here one can see the sudden growth of error after several steps have been taken. The broken lines and circles represent the values determined with $\omega(0,0.01) = 1.0004883$, and the solid lines are the results without deliberate error. The differences between the data in the two cases are generally too small to appear on the plot until $s = 0.112$. From this line onward, the growth is very fast. As mentioned previously, quantities on the line $s = 0.176$ could not be determined because of overflow in the calculator.

The following table of values of the modified stream function ω and of the second derivative ω_{ss} will give a more precise picture of the various effects. The numbers are all rounded to four significant figures for ease of perusal, except for the initial values of ω where the errors would not then be apparent.

s	$\omega[s,(1/2)\Delta t]$					
	$\Delta s = 0.032$			$\Delta s = 0.016$		
0	1.0000000	1.0000010	1.0004883	1.0000000	1.0000010	1.0004883
.032	.8846	.8846	.8851	.8845	.8845	.8849
.064	.7657	.7658	.7660	.7650	.7650	.7647
.096	.6407	.6407	.6394	.6384	.6384	.6330
.128	.5053	.5053	.4977	.5004	.5004	.4660
.160	.3541	.3540	.3220	.3446	.3443	.1565
.192	.1788	.1786	.05742	.1617	.1600	---
.224	-.03299	-.03379	-.5173	-.06576	-.07005	---
ω_{ss}						
0.032	-4.432	-4.433	-4.885	-4.398	-4.400	-5.359
.064	-7.732	-7.737	-9.872	-7.687	-7.697	-12.78
.096	-12.35	-12.36	-19.48	-12.37	-12.42	-39.82
.128	-18.77	-18.83	-47.10	-19.02	-19.28	-162.8
.160	-28.04	-28.21	-126.4	-28.93	-30.15	-785.3
.192	-43.41	-44.04	-479.4	-47.08	-50.67	---
.224	-24.90	-31.15	-1113	-96.54	-131.1	---

Effect of Error at Shock on p , ρ , ρ_s

$\Delta s = 0.032$									
s	$\omega(0,0.01) = 1.0000000$			$\omega(0,0.01) = 1.0000010$			$\omega(0,0.01) = 1.0004883$		
	ρ	ρ_s	p	ρ	ρ_s	p	ρ	ρ_s	p
0.032	4.6820	3.114	0.8539	4.6820	3.114	0.8539	4.6821	3.117	0.8540
.064	4.7734	2.597	.8774	4.7734	2.597	.8774	4.7737	2.606	.8775
.096	4.8498	2.179	.8971	4.8498	2.179	.8971	4.8506	2.202	.8973
.128	4.9134	1.796	.9136	4.9134	1.796	.9136	4.9152	1.835	.9141
.160	4.9641	1.372	.9269	4.9641	1.372	.9269	4.9661	1.349	.9274
.192	4.9987	.7873	.9359	4.9986	.7863	.9359	4.9835	-.2640	.9320
.224	5.0080	-.2009	.9384	5.0079	-.2087	.9383	4.7444	-14.68	.8699
$\Delta s = 0.016$									
.032	4.6819	3.133	.8539	4.6819	3.133	.8539	4.6820	3.138	.8540
.064	4.7738	2.628	.8775	4.7738	2.628	.8775	4.7743	2.650	.8776
.096	4.8512	2.217	.8975	4.8512	2.217	.8975	4.8532	2.297	.8980
.128	4.9160	1.833	.9143	4.9160	1.833	.9143	4.9224	2.018	.9160
.160	4.9678	1.392	.9279	4.9679	1.392	.9279	4.9633	-.1636	.9267
.192	5.0027	.7509	.9370	5.0027	.7433	.9370			
.224	5.0095	-.4437	.9388	5.0086	-.4888	.9385			

The enormous growth of error is quite apparent when the second derivative ω_{ss} is inspected. It is to be noted that halving the interval size has an effect on the original solution that is numerically greater than that of the artificial error of 1×10^{-6} and less than that of the artificial error of 5×10^{-4} . A tabulation of the values of density ρ , the derivative ρ_s , and pressure p , is also shown. Here the error is much less striking, probably due to the external iteration performed on ρ . From this example, it would appear worthwhile to try iteration on the stream function also in future work.

In an attempt to offset the inherent instability discussed above, a smoothing process was inserted in the machine program. For simplicity, an available SHARE routine was used.¹ This process causes a recalculation of the numerical values of the flow quantities along each $s = \text{constant}$ line by means of a least-square fit of a third-degree polynomial to seven consecutive points. Wild points are discarded and replaced by calculated values, and numerical differentiation based on a three-point formula is then performed. This routine is actually somewhat more elaborate than is necessary for the present purposes, and a heavy penalty is paid in computing time as a result. When the program runs without the smoothing procedure, seven-point numerical differentiation formulas are used for the derivatives with respect to t . In this latter instance, a single

¹The SHARE program in question is referenced as CL SMD3, SHARE distribution no. 331, entitled "Smooth and Differentiate Unequally Spaced Points."

case is completed in about 1 minute on an IBM 704 computer, but with the smoothing procedure operative, the average case runs 15 to 20 minutes.

In order to assess the effect of the smoothing operation inserted in the program, the case discussed above was run at several intervals. It should be mentioned that the unsmoothed program, in which seven-point differentiation with respect to t is used, is set to drop the three values of the dependent variables corresponding to the three highest values of t at each step. This is done to keep central difference formulas applicable throughout, because it was believed that uncertainties propagated from these top positions were a major cause for error growth. Thus, it is not possible to use a very small interval in connection with the unsmoothed program, for the number of steps to the body is limited to 15 by the point-dropping. This limitation does not exist in the program with smoothing.

A comparison of the values of the modified stream function ω as calculated using the smoothed and unsmoothed programs is shown in figure 2. Figure 2(a) shows the variation of $\omega[s, (1/2)\Delta t]$ with s for several values of Δs , and for runs made with and without smoothing. In each case, the effect of the smoothing is to raise the level of ω over the entire course of its variation, without affecting the general form of the curve. This is due to smaller numerical values of the second derivative ω_{ss} being found in the smoothed case. It does not seem possible to say whether this is affecting the solution process in a proper way or not, but it does appear that the closeness of the results in cases run with and without smoothing and the established tendency of the unsmoothed solution toward enlargement of errors make the likelihood of an adverse effect by the smoothing process rather small. A table of values of ω for the various cases is included so that a closer comparison can be made.

s	ω				
	Smoothed			Unsmoothed	
	$\Delta s = 0.032$	$\Delta s = 0.016$	$\Delta s = 0.008$	$\Delta s = 0.032$	$\Delta s = 0.016$
0	1.0000	1.0000	1.0000	1.0000	1.0000
.032	.8846	.8845	.8844	.8846	.8845
.064	.7658	.7650	.7646	.7658	.7650
.096	.6409	.6388	.6375	.6407	.6384
.128	.5061	.5015	.4988	.5054	.5004
.160	.3559	.3473	.3421	.3541	.3446
.192	.1830	.1681	.1591	.1788	.1617
.224	-.02382	-.04804	-.06240	-.03299	-.04804

In figure 2(b), stagnation pressure p_{st} and stand-off distance Δ/R_s are plotted versus interval size for several smoothed and unsmoothed runs. (It should be mentioned that in the smoothed calculation, the entropy function $f(\psi)$ is fixed at its known value on the body when pressures on the body are computed.) The dashed line on the pressure plot represents the known exact value of stagnation pressure. Two additional smoothed runs were made, with $\Delta s = 0.008$ and 0.010 (the case with $\Delta s = 0.008$ also appears in fig. 2(a)), and the results were plotted in figure 2(b). The plot of stagnation pressures against interval size Δs seems rather inconclusive since sufficiently small intervals for comparison are not available in the unsmoothed case. However, the runs with smoothing do approach the known correct value in a rather consistent manner.

In the case of the plot of detachment distance Δ/R_s versus Δs , it is noted that the smoothed points lie on a line so that linear extrapolation to the limit of vanishing interval size (in the s direction) is possible. On the other hand, the unsmoothed points for Δ/R_s lie on a line until the interval size Δs becomes 0.015 , and this is what one would expect; that is, the solutions will begin to be less accurate as the number of steps taken becomes too large. Unfortunately, smaller values of Δs cannot be used in the unsmoothed program because the body streamline would not be reached. The nearly constant difference in level between the smoothed and unsmoothed extrapolation lines follows from the properties of these solutions mentioned above in the discussion of figure 2(a). Smaller intervals for the smoothed case were not run because of the large amount of machine time used. For $\Delta s = 0.008$, 26 steps were taken to reach the body.

Several runs were made with the smoothed program in which a deliberate error was forced in the initial data. Figure 3(a) shows plots of $\omega(s, t)$ versus t at several values of s proceeding from the shock toward the body. The error in initial data was very large, the value of ω being taken as 1.1 at $s = 0$, and $t = 0.01$. The propagation of the error can be followed along the successive $s = \text{constant}$ lines in figure 3(a), and its spread and intensification are fairly rapid. The error is, of course, enormous for a round-off error, but its large size makes the effect easier to follow. Figures 3(b) and 3(c) show ω as a function of s for $t = 0.01$ and $t = 0.21$, respectively. The effect of errors in $\omega(0, 0.01)$ of 0.1 and 0.01 are shown in figure 3(b), while only the negligible effect of the larger error on $\omega(s, 0.21)$ is shown in figure 3(c). A deliberate error introduced at a higher value of t , for example, $t = 0.11$, has a much smaller effect on the solution, as was remarked by Van Dyke in reference 1. The values above and below $t = 0.11$ are apparently unaffected, and the bump in the data merely propagates across to the body, being lightly damped rather than growing.

Finally, it is the author's opinion that the use of smoothing in the numerical solution does not affect the results adversely, but has the effect of allowing the use of a smaller interval size than is feasible with an unsmoothed program. It should be stated, however, that the

instability of the set of equations (9) is not as serious as might have been expected. This may be because the end points of each $s = \text{constant}$ line are dropped, to avoid error which could arise from the noncentral derivative formula. There is also a strong possibility that the nonlinearity of equations (9) has an alleviating effect on the instability. In this latter connection it is interesting to note that the present method of numerical solution was attempted for an incompressible flow about a circular cylinder, where the governing equation is Laplace's. It was found that the instability was disastrous in this case, even when the data were subjected to smoothing. Thus, some connection might be made between the "strength of the ellipticity" of an equation and its degree of instability toward random error when used in connection with boundary conditions of Cauchy type.

NUMERICAL RESULTS

As an illustration of the dependence of such geometric quantities as Δ/R_s , Δ/R_b , R_b/R_s , and B_b on the shock parameter B_s , several cases were run off at $M = 4$ and $\gamma = 1.4$ with $-2 \leq B_s \leq +2$. The results are summarized in figure 4. The bodies resulting from the various values of B_s were not in all instances well fit by conic sections. This was particularly true for the smaller values of B_s . However, in these cases, the stagnation region was reasonably well approximated by the conic section so that the results hold at least for this portion of the bodies.

The additional geometric parameter A , introduced in equation (2a), has the effect of making very blunt bodies available. Figure 5 illustrates this for a free stream at $M = 10$ and $\gamma = 1.4$. The body corresponding to $A = 1$ is actually slightly concave near the stagnation point. The $A = 0$ body is a circular cylinder, and the $A = -1$ body is nearly parabolic. In the latter case, the sonic point on the body was not reached. In fact, sonic speed behind the shock was not attained on the portion shown in figure 5. The pressure distributions at the body surfaces for the various cases are also shown, plotted against the vertical coordinate y .

The variation with γ of the geometric parameters of a body corresponding to a parabolic shock wave with infinite free-stream Mach number is shown in figure 6 where the shock wave and the various bodies are plotted. For $\gamma = 1$ at $M = \infty$, the body and shock are coincident. The calculations were done for $\gamma = 1.05, 1.1, 1.2, 1.4, 1.6667, 2$, and 7 . The table insert in figure 6(a) gives the numerical values of the geometric parameters associated with the several bodies. In addition, the variations of Δ/R_s and R_b/R_s are plotted against $(\gamma - 1)/(\gamma + 1)$. From the results shown in figure 6, one can see that the parameter B_b is not enough to specify a shape; the radius R_b must also be given.

Flow Fields About Circular Cylinders

The main results of the present calculations are contained in table I, where some characteristics of flow about circular cylinders at various free-stream Mach numbers and with several values of the adiabatic index γ are listed. The Mach numbers used are $M = \infty, 20, 10, 8, 6, 4, 3$, and 2, and for each of these, the parameter γ takes the values $6/5$, $7/5$, and $5/3$. Quantities given for each set of conditions are the coordinates of shock, body and sonic line, and pressures on the body.

It did not prove feasible to give results for free-stream Mach numbers less than 2 for these two-dimensional flows. The greater stand-off distances and generally large extent (based on body size) of the subsonic field in plane flows as opposed to axially symmetric flows make determination at low Mach numbers very difficult. If one confined attention to bodies rather blunter than a circular cylinder, solutions could probably be found down to Mach numbers of about 1.2, but precise results in this range are not presently available. For the $M = 2$ cases with $\gamma = 7/5$ and $5/3$, a well-established sonic line was not found. This is presumably because of its large extent through the flow field and consequent sensitivity to errors made at the largest values of the vertical coordinate t .

Most of the solutions listed in table I were found using the smoothing process discussed above. However, some cases were determined before this process was incorporated. Spot checks were made to compare the results with and without smoothing, and the closeness thereof in the cases checked gave confidence in using the unsmoothed results where available. In all cases, the experimental runs to determine the approximate value of B_s at a given M and γ were made with the unsmoothed process because of the much greater speed of that program.

The pressure ratios given in table I are the ratios of local pressure to stagnation pressure p_{st} at the apex of the body. The values of p_{st} used are those given by the numerical solution, and are listed for each case, along with the known exact values computed from (see ref. 9)

$$p_{st} = \frac{1}{\gamma M^2} \left(\frac{\gamma + 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \left[\frac{\gamma + 1}{2\gamma M^2 - (\gamma - 1)} \right]^{\frac{1}{\gamma - 1}}$$

The calculated values of pressure ratio at $M = \infty$ may be compared with values given by the "modified Newtonian" pressure formula, namely

$$\frac{p}{p_{st}} = \sin^2 \theta_b \quad (19)$$

where θ_b is the angle made with the stream direction by a tangent to the body surface. (For a discussion of Newtonian results, see ref. 4.) The comparison (see fig. 7) shows fair agreement between the calculated values and those given by equation (19).

The fortuitous agreement between modified Newtonian and actual pressure distributions at $\gamma = 1.4$ is demonstrated, though in lesser degree than in axially symmetric flow. The behavior of the pressure distributions for decreasing γ is somewhat surprising in that for $\gamma = 5/3$, $7/5$, and $6/5$, the several pressure distributions fall first below, then above the modified Newtonian result. This is seemingly the wrong trend, for as $\gamma \rightarrow 1$, the limiting pressure distribution should be the Newton-Busemann one, where the centrifugal correction has been made. The formula for this is

$$\frac{p}{p_{st}} = \sin^2 \theta_b - \frac{1}{2} \cos^2 \theta_b$$

(see ref. 4), and a plot of it is also shown in figure 7. In order to determine whether the trend is not yet established because $\gamma = 1.2$ is still too far from $\gamma = 1$, additional solutions were run for a circular cylinder at infinite Mach number and for $\gamma = 1.1$ and 1.05 . The results from these runs are plotted in figure 7, and one can see that they do indeed fall as they should. Thus, it is seen that γ must be quite near to unity in order that the limiting case of Newtonian flow be a fair representation of the actual results.

The results for $M = 20$ fall very nearly as those for $M = \infty$, and results from a version of equation (19) corrected for finite Mach number

$$\frac{p}{p_{st}} = \frac{1 + \gamma M^2 \sin^2 \theta_b}{1 + \gamma M^2} \quad (19a)$$

hardly differ from those for $M = \infty$. For the lower Mach numbers, however, agreement between calculated and modified Newtonian results becomes increasingly poor, the calculated results being generally the higher so that the centrifugal correction would only worsen the situation.

Unfortunately, the data for two-dimensional blunt-body flows are not so plentiful as for axisymmetric ones. However, some numerical solutions are available in references 10 and 11, and reference 12 gives a compilation of experimental data. Standoff distances from these references are plotted with the present results in figure 8. The results of reference 10 and the present ones differ from the experimental data of reference 12 by about equal amounts but in opposite directions. The asymptotic value given in reference 11 is matched very closely by the present result, calculated for $M = 10^4$.

The results for $\gamma = 1.4$ found here also correlate well on the basis of the "detachment angle" parameters discussed in reference 13.

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A
3
7
2

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS
(a) $\gamma = 1.4$

M = ∞						
$B_S = 0.1$; $R_b/R_S = 0.5003$; $\Delta/R_S = 0.1888$; $p_{st} = 0.9193$ (0.9197)						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	p/p_{st}	x	y
0	0	0.1888	0	1.000	0.0825	0.405
.03	.00045	.1897	3.44	.996	.12	.430
.07	.00245	.1937	8.05	.979	.16	.449
.11	.00605	.2011	12.7	.949	.20	.459
.15	.01126	.2118	17.4	.906	.24	.456
.19	.01807	.2262	22.3	.852	.28	.422
.23	.02649	.2447	27.4	.786	.30	.381
.27	.03652	.2678	32.7	.710	.31	.348
.31	.04817	.2964	38.3	.625	---	---
.35	.06144	.3318	44.4	.529	---	---
.39	.07634	.3776	51.2	.417	---	---
.43	.09288	---	---	---	---	---
M = 20						
$B_S = 0.16$; $R_b/R_S = 0.4991$; $\Delta/R_S = 0.1908$; $p_{st} = 0.92062$ (0.92051)						
0	0	.1908	0	1.000	.0830	.405
.03	.00045	.1917	3.4	.996	.12	.439
.07	.002450	.1957	8.1	.979	.16	.447
.11	.006053	.2030	12.7	.950	.20	.455
.15	.01126	.2137	17.5	.907	.24	.450
.19	.01808	.2281	22.4	.852	.28	.406
.23	.02651	.2465	27.4	.786	.30	.339
.27	.03656	.2697	32.8	.708	---	---
.31	.04824	.2986	38.4	.618	---	---
.35	.06155	.3347	44.5	.514	---	---
.39	.07652	.3826	51.4	.389	---	---
.43	.09314	---	---	---	---	---
M = 10						
$B_S = 0.1$; $R_b/R_S = 0.4860$; $\Delta/R_S = 0.1942$; $p_{st} = 0.92299$ (0.92298)						
0	0	.1942	0	1.000	.0860	.414
.015	.000113	.1945	1.8	.999	.12	.434
.045	.001013	.1963	5.3	.991	.16	.453
.075	.002813	.2001	8.9	.975	.20	.459
.105	.005514	.2057	12.5	.952	.24	.453
.135	.009117	.2133	16.1	.921	.28	.417
.165	.01362	.2230	19.8	.883	.30	.384
.195	.01903	.2349	23.6	.839	.32	.339
.225	.02534	.2492	27.6	.788	---	---
.255	.03257	.2662	31.6	.731	---	---
.285	.04070	.2863	35.9	.667	---	---
.315	.04974	.3099	40.4	.598	---	---
.345	.05969	.3381	45.2	.521	---	---
.375	.07056	.3722	50.5	.436	---	---
.405	.08235	.4152	56.4	.338	---	---
.435	.09506	---	---	---	---	---

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS - Continued
(a) $\gamma = 1.4$ - Continued

M = 8						
$B_s = 0.1$; $R_b/R_s = 0.4784$; $\Delta/R_s = 0.1969$; $p_{st} = 0.92533$ (0.92484)						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	p/p_{st}	x	y
0	0	0.1969	0	1.000	0.0880	0.418
.03	.0005	.1978	3.6	.996	.12	.437
.07	.0025	.2020	8.4	.978	.16	.454
.11	.0061	.2097	13.3	.946	.20	.462
.15	.0113	.2210	18.3	.901	.24	.452
.19	.0181	.2362	23.4	.843	.28	.415
.23	.0265	.2557	28.7	.773	.30	.366
.27	.0365	.2803	34.4	.691	---	---
.31	.0482	.3111	40.4	.598	---	---
.35	.0614	.3499	47.0	.491	---	---
.39	.0763	.4011	54.6	.367	---	---
.43	.0929	---	---	---	---	---
M = 6						
$B_s = 0$; $R_b/R_s = 0.4558$; $\Delta/R_s = 0.2026$; $p_{st} = 0.9283$ (0.9289)						
0	0	.2026	0	1.000	.094	.433
.03	.0004	.2036	3.8	.996	.12	.447
.07	.0024	.2081	8.8	.976	.16	.461
.11	.0060	.2161	14.0	.942	.20	.467
.15	.0112	.2279	19.2	.893	.24	.460
.19	.0180	.2439	24.6	.830	.28	.427
.23	.0264	.2646	30.3	.755	.30	.384
.27	.0364	.2906	36.3	.668	---	---
.31	.0480	.3234	42.8	.569	---	---
.35	.0612	.3653	50.2	.459	---	---
.39	.0760	---	---	---	---	---
.43	.0924	---	---	---	---	---
M = 4						
$B_s = -0.06$; $R_b/R_s = 0.4151$; $\Delta/R_s = 0.2182$; $p_{st} = 0.94123$ (0.94054)						
0	0	.2182	0	1.000	.107	.464
.03	.0004	.2193	4.15	.995	.12	.468
.07	.0024	.2242	9.71	.973	.16	.476
.11	.0060	.2330	15.4	.933	.20	.477
.15	.0112	.2462	21.2	.877	.24	.458
.19	.0180	.2642	27.2	.806	.28	.409
.23	.0264	.2876	33.6	.719	.30	.356
.27	.0364	.3178	40.6	.618	---	---
.31	.0480	.3570	48.3	.501	---	---
.35	.0611	---	---	---	---	---
.39	.0759	---	---	---	---	---
.43	.0922	---	---	---	---	---
.47	.1101	---	---	---	---	---

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS - Continued
(a) $\gamma = 1.4$ - Concluded

M = 3						
$B_s = -0.26$; $R_b/R_s = 0.3602$; $\Delta/R_s = 0.2362$; $p_{st} = 0.9577$ (0.95722)						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	p/p_{st}	x	y
0	0	0.2362	0	1.000	0.127	0.509
.03	.0004	.2374	4.78	.994	.16	.508
.07	.0024	.2430	11.2	.966	.20	.498
.11	.0060	.2533	17.8	.916	.24	.469
.15	.0112	.2688	24.6	.844	.28	.403
.19	.0180	.2903	31.8	.753	.30	.330
.23	.0264	.3191	39.7	.641	---	---
.27	.0363	.3579	48.6	.510	---	---
.31	.0478	---	---	---	---	---
.35	.0608	---	---	---	---	---
.39	.0753	---	---	---	---	---
.43	.0914	---	---	---	---	---
.47	.1089	---	---	---	---	---
.51	.1279	---	---	---	---	---
M = 2						
$B_s = -1.5$; $R_b/R_s = 0.2287$; $\Delta/R_s = 0.2661$; $p_{st} = 1.0062$ (1.0072)						
0	0	.2661	0	1.000	.226	.728
.03	.0004	.2680	7.54	.986	.24	.710
.07	.0024	.2770	17.8	.924	.28	.652
.11	.0060	.2942	28.8	.811	.32	.586
.15	.0112	.3222	41.0	.644	.36	.495
.19	.0178	.3679	56.2	.426	.38	.433
.23	.0259	---	---	---	---	---
.27	.0355	---	---	---	---	---
.31	.0464	---	---	---	---	---
.35	.0587	---	---	---	---	---
.39	.0721	---	---	---	---	---
.43	.0868	---	---	---	---	---
.47	.1026	---	---	---	---	---
.51	.1194	---	---	---	---	---
.55	.1371	---	---	---	---	---
.59	.1558	---	---	---	---	---
.63	.1754	---	---	---	---	---
.67	.1957	---	---	---	---	---
.71	.2168	---	---	---	---	---
.75	.2386	---	---	---	---	---

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS - Continued
(b) $\gamma = 1.2$

M = ∞						
$B_S = 0.12$; $R_b/R_S = 0.6208$; $\Delta/R_S = 0.1262$; $p_{st} = 0.95557$ (0.95550)						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	p/p_{st}	x	y
0	0	0.1262	0	1.000	0.045	0.300
.025	.0003	.1267	2.31	.998	.08	.343
.065	.0021	.1296	6.01	.988	.12	.386
.105	.0055	.1352	9.73	.969	.16	.422
.145	.0105	.1434	13.5	.941	.20	.448
.185	.0171	.1545	17.3	.906	.24	.459
.225	.0254	.1685	21.2	.863	.28	.432
.265	.0352	.1856	25.3	.813	---	---
.305	.0466	.2059	29.4	.757	---	---
.345	.0597	.2302	33.8	.696	---	---
.385	.0744	.2588	38.3	.629	---	---
.425	.0908	.2928	43.2	.554	---	---
.465	.1088	.3347	49.3	.460	---	---
M = 20						
$B_S = 0.114$; $R_b/R_S = 0.6049$; $\Delta/R_S = 0.1280$; $p_{st} = 0.95675$ (0.95650)						
0	0	.1280	0	1.000	.046	.303
.03	.0005	.1288	2.84	.997	.08	.345
.07	.0025	.1321	6.65	.986	.12	.387
.11	.0061	.1381	10.5	.965	.16	.422
.15	.0113	.1469	14.4	.936	.20	.446
.19	.0181	.1586	18.3	.898	.24	.458
.23	.0265	.1733	22.3	.854	.28	.439
.27	.0365	.1914	26.5	.802	---	---
.31	.0482	.2132	30.8	.745	---	---
.35	.0615	.2385	35.4	.676	---	---
.39	.0764	.2690	40.1	.607	---	---
.43	.0929	.3057	45.3	.525	---	---
.47	.1111	.3501	51.0	.433	---	---
M = 10						
$B_S = 0.11$; $R_b/R_S = 0.5904$; $\Delta/R_S = 0.1342$; $p_{st} = 0.95948$ (0.95949)						
0	0	.1342	0	1.000	.049	.314
.015	.0001	.1344	1.46	.999	.08	.350
.045	.0010	.1359	4.37	.994	.12	.389
.075	.0028	.1389	7.29	.980	.16	.421
.105	.0055	.1435	10.2	.967	.20	.444
.135	.0091	.1497	13.2	.945	.24	.451
.165	.0136	.1575	16.2	.920	.28	.419
.195	.0190	.1670	19.3	.890	---	---
.225	.0253	.1782	22.4	.855	---	---
.255	.0326	.1913	25.6	.815	---	---
.285	.0407	.2064	28.9	.774	---	---
.315	.0497	.2237	32.2	.727	---	---
.345	.0597	.2435	35.8	.679	---	---
.375	.0706	.2660	39.4	.624	---	---
.405	.0824	.2919	43.4	.567	---	---
.435	.0951	.3222	47.5	.500	---	---
.465	.1088	.3591	52.0	.423	---	---

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS - Continued
(b) $\gamma = 1.2$ - Continued

M = 8						
$B_S = 0.06$; $R_b/R_S = 0.5743$; $\Delta/R_S = 0.1384$; $p_{st} = 0.96170$ (0.96175)						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	p/p_{st}	x	y
0	0	0.1384	0	1.000	0.052	0.323
.03	.0005	.1392	2.99	.997	.08	.352
.07	.0025	.1427	7.00	.985	.12	.391
.11	.0061	.1491	11.0	.963	.16	.422
.15	.0113	.1584	15.1	.932	.20	.444
.19	.0181	.1708	19.3	.893	.24	.452
.23	.0265	.1864	23.6	.846	.28	.431
.27	.0365	.2057	28.0	.790	.30	.404
.31	.0481	.2289	32.7	.726	---	---
.35	.0614	.2568	37.6	.659	---	---
.39	.0762	.2907	42.8	.587	---	---
.43	.0927	.3322	48.5	.503	---	---
M = 6						
$B_S = 0.106$; $R_b/R_S = 0.5555$; $\Delta/R_S = 0.1474$; $p_{st} = 0.96670$ (0.96664)						
0	0	.1474	0	1.000	.057	.338
.015	.0001	.1475	1.55	.999	.08	.360
.045	.0010	.1492	4.65	.994	.12	.394
.075	.0028	.1524	7.76	.983	.16	.421
.105	.0055	.1573	10.9	.964	.20	.438
.135	.0091	.1639	14.1	.940	.24	.438
.165	.0136	.1722	17.3	.910	.28	.391
.195	.0190	.1824	20.5	.878	---	---
.225	.0253	.1945	23.9	.841	---	---
.255	.0326	.2087	27.3	.797	---	---
.285	.0407	.2250	30.9	.752	---	---
.315	.0497	.2443	34.5	.699	---	---
.345	.0597	.2663	38.4	.643	---	---
.375	.0706	.2918	42.5	.582	---	---
.405	.0824	.3218	46.8	.510	---	---
.435	.0951	---	---	---	---	---
M = 4						
$B_S = 0$; $R_b/R_S = 0.4936$; $\Delta/R_S = 0.1711$; $p_{st} = 0.9801$ (0.98078)						
0	0	.1711	0	1.000	.073	.381
.03	.0004	.1720	3.48	.997	.12	.412
.07	.0024	.1761	8.15	.982	.16	.430
.11	.0060	.1835	12.9	.954	.20	.437
.15	.0112	.1944	17.7	.914	.24	.428
.19	.0180	.2091	22.6	.864	.28	.376
.23	.0264	.2278	27.8	.802	---	---
.27	.0364	.2513	33.2	.731	---	---
.31	.0480	.2804	38.9	.649	---	---
.35	.0612	.3168	45.2	.556	---	---
.39	.0760	---	---	---	---	---
.43	.0924	---	---	---	---	---

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS - Continued
(b) $\gamma = 1.2$ - Concluded

M = 3						
$B_s = -0.13$; $R_b/R_s = 0.4238$; $\Delta/R_s = 0.1977$; $p_{st} = 1.0017$ (1.0010)						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	p/p_{st}	x	y
0	0	0.1977	0	1.000	0.093	0.433
.03	.0004	.1988	4.06	.996	.12	.442
.07	.0024	.2035	9.51	.976	.16	.448
.11	.0060	.2122	15.0	.940	.20	.442
.15	.0112	.2251	20.7	.890	.24	.418
.19	.0180	.2427	26.6	.825	.28	.341
.23	.0264	.2656	32.9	.746	---	---
.27	.0364	.2952	39.6	.652	---	---
.31	.0479	.3336	47.0	.541	---	---
.35	.0610	---	---	---	---	---
.39	.0757	---	---	---	---	---
.43	.0919	---	---	---	---	---
.47	.1097	---	---	---	---	---
M = 2						
$B_s = -1.15$; $R_b/R_s = 0.2652$; $\Delta/R_s = 0.2454$; $p_{st} = 1.0607$ (1.0616)						
0	0	.2454	0	1.000	.168	.606
.03	.0004	.2471	6.49	.990	.20	.577
.07	.0024	.2548	15.3	.946	.24	.531
.11	.0060	.2692	24.5	.866	.28	.445
.15	.0112	.2919	34.4	.748	.30	.211
.19	.0179	.3262	45.8	.591	.31	.173
.23	.0261	.3795	60.1	.396	---	---
.27	.0357	---	---	---	---	---
.31	.0468	---	---	---	---	---
.35	.0592	---	---	---	---	---
.39	.0730	---	---	---	---	---
.43	.0880	---	---	---	---	---
.47	.1042	---	---	---	---	---
.51	.1216	---	---	---	---	---
.55	.1400	---	---	---	---	---
.59	.1594	---	---	---	---	---
.63	.1799	---	---	---	---	---

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS - Continued
(c) $\gamma = 5/3$

$M = \infty$						
$B_s = 0.10; R_b/R_s = 0.4156; \Delta/R_s = 0.2428; p_{st} = 0.8805 (0.88132)$						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	p/p_{st}	x	y
0	0	0.2428	0	1.000	0.123	0.494
.03	.0005	.2439	4.14	.995	.16	.503
.07	.0025	.2487	9.69	.969	.20	.504
.11	.0061	.2576	15.3	.924	.24	.493
.15	.0113	.2707	21.2	.860	.28	.456
.19	.0181	.2887	27.2	.776	.32	.348
.23	.0265	.3123	33.6	.674	.34	.267
.27	.0365	.3430	40.5	.553	---	---
.31	.0482	.3840	48.2	.411	---	---
.35	.0614	---	---	---	---	---
.39	.0763	---	---	---	---	---
.43	.0929	---	---	---	---	---
.47	.1111	---	---	---	---	---
.51	.1309	---	---	---	---	---
$M = 20$						
$B_s = 0.08; R_b/R_s = 0.4116; \Delta/R_s = 0.2435; p_{st} = 0.8812 (0.88198)$						
0	0	.2435	0	1.000	.124	.496
.03	.0005	.2446	4.18	.994	.16	.505
.07	.0025	.2495	9.79	.969	.20	.506
.11	.0061	.2585	15.5	.923	.24	.494
.15	.0113	.2718	21.4	.858	.28	.438
.19	.0181	.2899	27.5	.774	.32	.352
.23	.0265	.3138	34.0	.671	.34	.272
.27	.0365	.3450	41.0	.550	---	---
.31	.0481	.3864	48.9	.409	---	---
.35	.0614	---	---	---	---	---
.39	.0763	---	---	---	---	---
.43	.0928	---	---	---	---	---
.47	.1109	---	---	---	---	---
.51	.1307	---	---	---	---	---
$M = 10$						
$B_s = 0.05; R_b/R_s = 0.4029; \Delta/R_s = 0.2460; p_{st} = 0.8833 (0.88397)$						
0	0	.2460	0	1.000	.127	.503
.03	.0005	.2471	4.27	.994	.16	.510
.07	.0025	.2521	10.0	.968	.20	.510
.11	.0061	.2613	15.8	.921	.24	.497
.15	.0113	.2749	21.9	.854	.28	.460
.19	.0181	.2935	28.1	.767	.32	.355
.23	.0265	.3181	34.8	.662	.34	.272
.27	.0365	.3503	42.1	.539	---	---
.31	.0481	.3433	50.3	.396	---	---
.35	.0613	---	---	---	---	---
.39	.0762	---	---	---	---	---
.43	.0927	---	---	---	---	---
.47	.1108	---	---	---	---	---
.51	.1305	---	---	---	---	---

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS - Continued
(c) $\gamma = 5/3$ - Continued

M = 8						
$B_S = 0$; $R_D/R_S = 0.3936$; $\Delta/R_S = 0.2476$; $p_{st} = 0.8849$ (0.88547)						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	P/P_{st}	x	y
0	0	0.2476	0	1.000	0.130	0.509
.03	.0004	.2487	4.37	.994	.16	.515
.07	.0024	.2538	10.2	.967	.20	.515
.11	.0060	.2632	16.2	.917	.24	.502
.15	.0112	.2772	22.4	.850	.28	.468
.19	.0180	.2963	28.9	.762	.32	.368
.23	.0264	.3216	35.8	.656	.34	.275
.27	.0364	.3547	43.3	.533	---	---
.31	.0480	.3990	52.0	.393	---	---
.35	.0612	---	---	---	---	---
.39	.0760	---	---	---	---	---
.43	.0924	---	---	---	---	---
.47	.1104	---	---	---	---	---
.51	.1300	---	---	---	---	---
M = 6						
$B_S = -0.02$; $R_D/R_S = 0.3831$; $\Delta/R_S = 0.2515$; $p_{st} = 0.8885$ (0.88871)						
0	0	.2515	0	1.000	.134	.518
.03	.0004	.2526	4.49	.994	.16	.522
.07	.0024	.2579	10.5	.965	.20	.518
.11	.0060	.2676	16.7	.915	.24	.504
.15	.0112	.2820	23.1	.844	.28	.463
.19	.0180	.3018	29.7	.752	.32	.349
.23	.0264	.3282	36.9	.641	.34	.261
.27	.0364	.3630	44.8	.511	---	---
.31	.0480	.4103	54.0	.362	---	---
.35	.0612	---	---	---	---	---
.39	.0760	---	---	---	---	---
.43	.0924	---	---	---	---	---
.47	.1103	---	---	---	---	---
.51	.1299	---	---	---	---	---
M = 4						
$B_S = -0.20$; $R_D/R_S = 0.3455$; $\Delta/R_S = 0.2608$; $p_{st} = 0.8970$ (0.89811)						
0	0	.2608	0	1.000	.149	.550
.03	.0004	.2621	4.98	.992	.16	.550
.07	.0024	.2680	11.7	.958	.20	.543
.11	.0060	.2787	18.6	.897	.24	.524
.15	.0112	.2949	25.7	.810	.28	.482
.19	.0180	.3175	33.4	.699	.32	.358
.23	.0264	.3484	41.7	.563	.34	.255
.27	.0363	.3912	51.4	.405	---	---
.31	.0478	---	---	---	---	---
.35	.0609	---	---	---	---	---
.39	.0755	---	---	---	---	---
.43	.0916	---	---	---	---	---
.47	.1093	---	---	---	---	---
.51	.1284	---	---	---	---	---
.55	.1490	---	---	---	---	---

TABLE I.- GEOMETRY AND PRESSURE DATA FOR CIRCULAR CYLINDERS - Concluded
(c) $\gamma = 5/3$ - Concluded

M = 3						
$B_s = -0.3$; $R_b/R_s = 0.3131$; $\Delta/R_s = 0.2728$; $p_{st} = 0.9116$ (0.91153)						
Shock		Body		Pressure ratio	Sonic line	
y	x	x	$90^\circ - \theta_b$	p/p _{st}	x	y
0	0	0.2728	0	1.000	0.164	0.581
.03	.0004	.2743	5.49	.991	.20	.566
.07	.0024	.2808	12.9	.950	.24	.534
.11	.0060	.2928	20.6	.878	.28	.468
.15	.0112	.3111	28.6	.774	.30	.393
.19	.0180	.3374	35.8	.637	.32	.280
.23	.0263	.3749	47.3	.465	.34	.208
.27	.0363	---	---	---	---	---
.31	.0477	---	---	---	---	---
.35	.0607	---	---	---	---	---
.39	.0752	---	---	---	---	---
.43	.0912	---	---	---	---	---
.47	.1087	---	---	---	---	---
.51	.1276	---	---	---	---	---
.55	.1480	---	---	---	---	---
.59	.1697	---	---	---	---	---
M = 2						
$B_s = -2.0$; $R_b/R_s = 0.1938$; $\Delta/R_s = 0.2849$; $p_{st} = 0.9504$ (0.9518)						
0	0	.2849	0	1.000	.367	1.00
.03	.0004	.2872	8.90	.980	.40	.940
.07	.0024	.2979	21.2	.888	.48	.675
.11	.0060	.3189	34.6	.716	---	---
.15	.0111	.3564	50.7	.456	---	---
.19	.0177	---	---	---	---	---
.23	.0258	---	---	---	---	---
.27	.0352	---	---	---	---	---
.31	.0459	---	---	---	---	---
.35	.0579	---	---	---	---	---
.45	.0927	---	---	---	---	---
.55	.1334	---	---	---	---	---
.65	.1792	---	---	---	---	---
.75	.2289	---	---	---	---	---
.85	.2818	---	---	---	---	---
.95	.3374	---	---	---	---	---
.99	.3603	---	---	---	---	---

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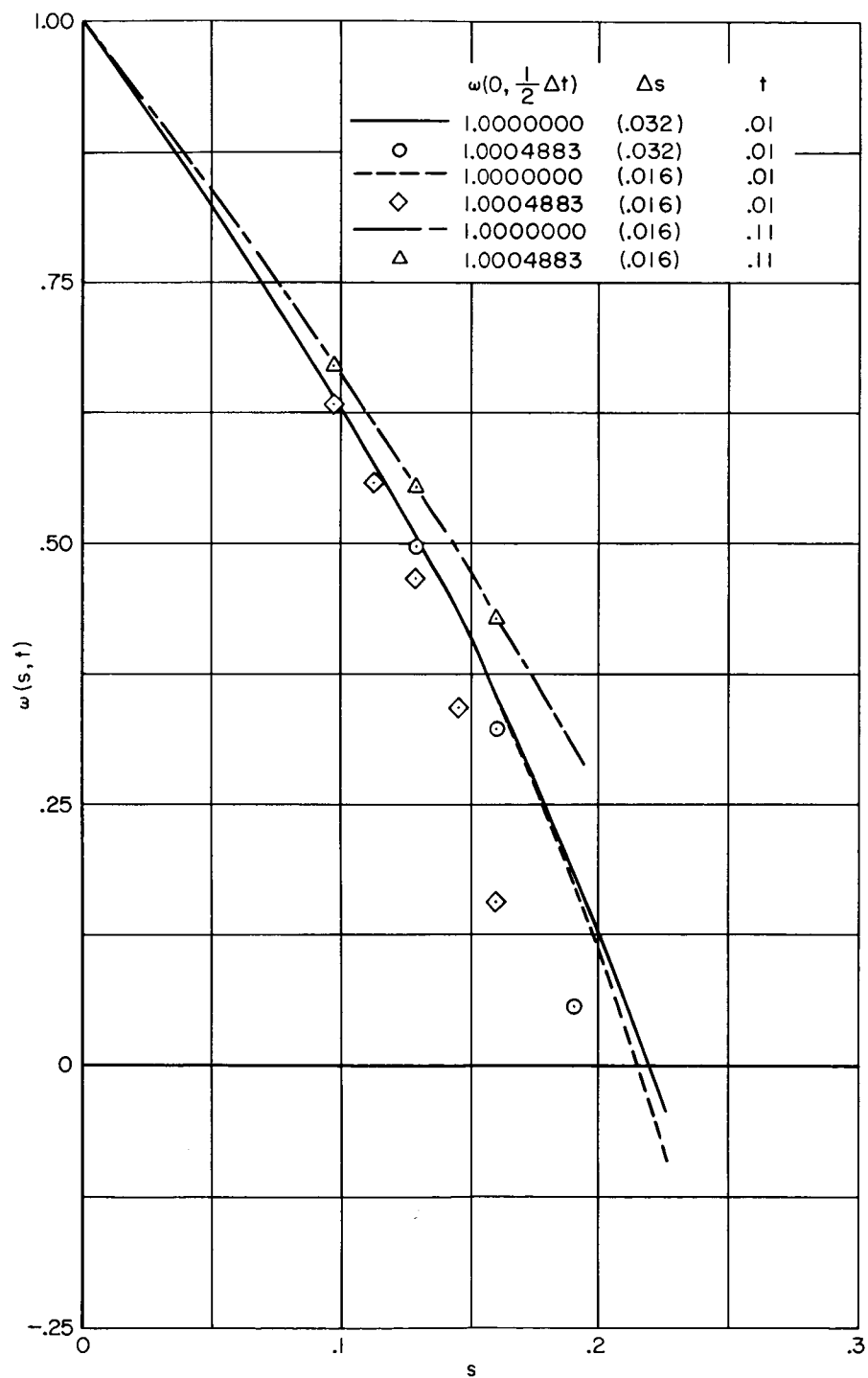
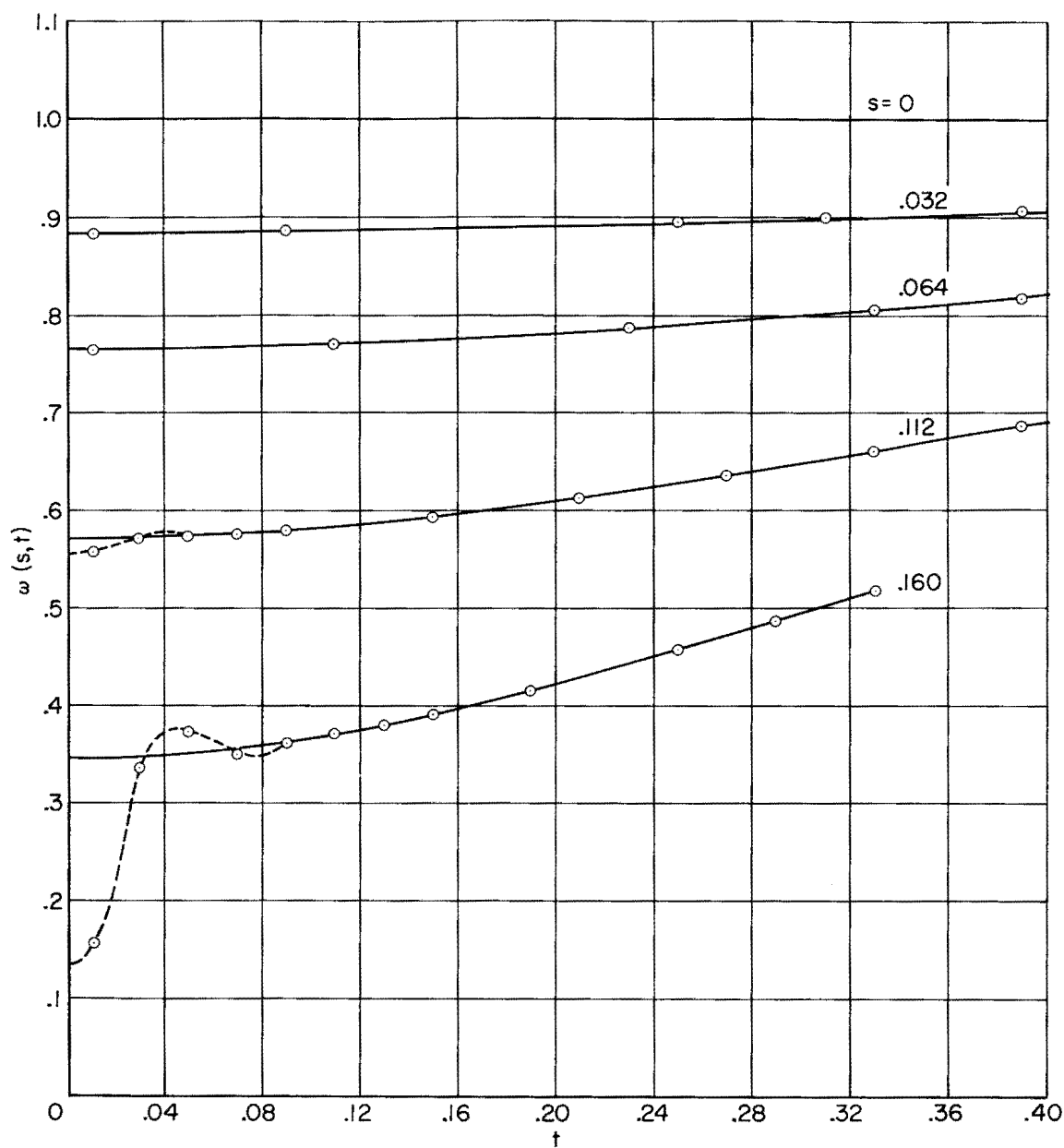
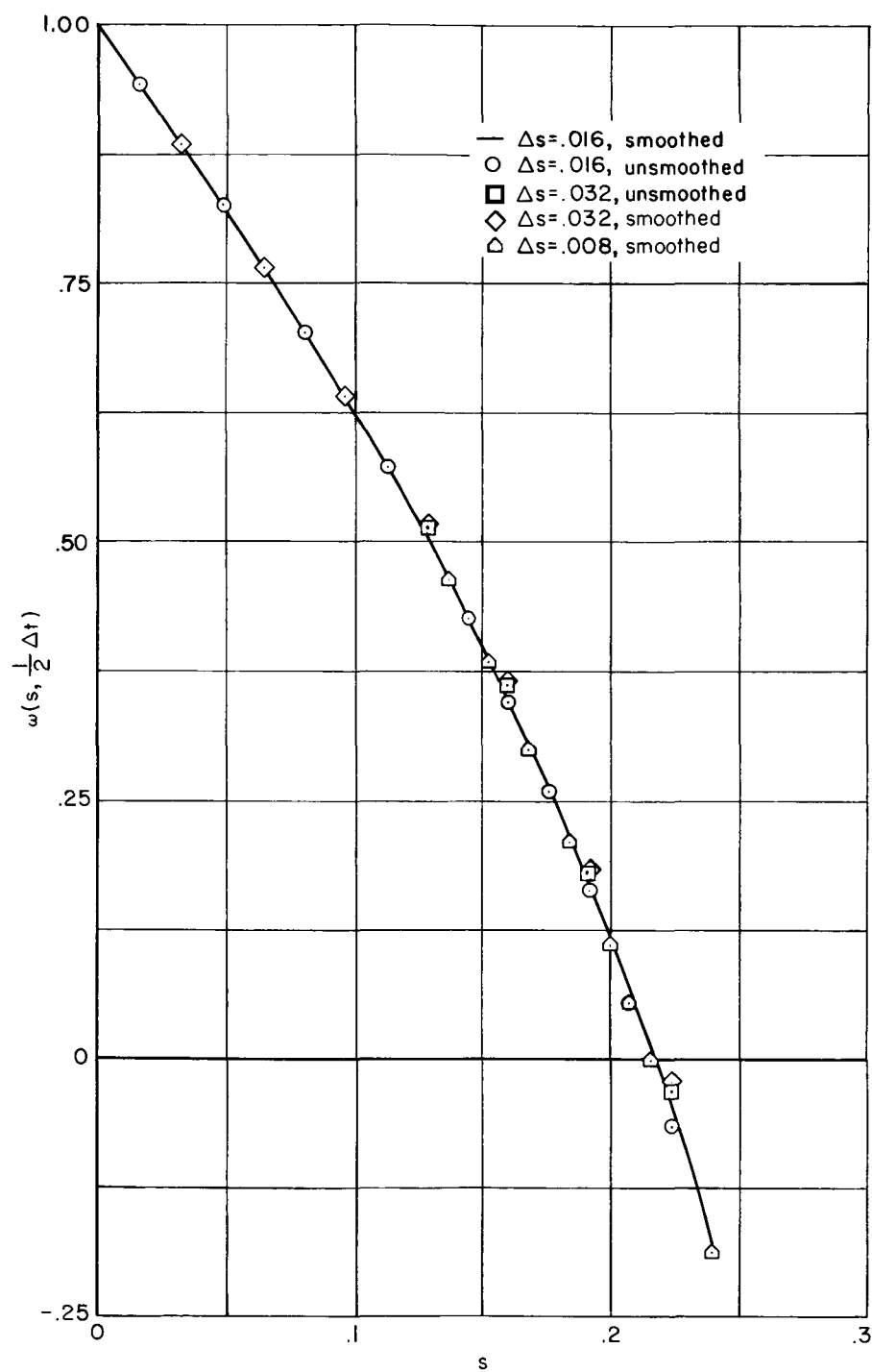
(a) $w(s, t)$ as function of s .

Figure 1.- Effect of artificial error on unsmoothed calculations;
 $M = 4$, $\gamma = 1.4$, $B_S = -0.06$.



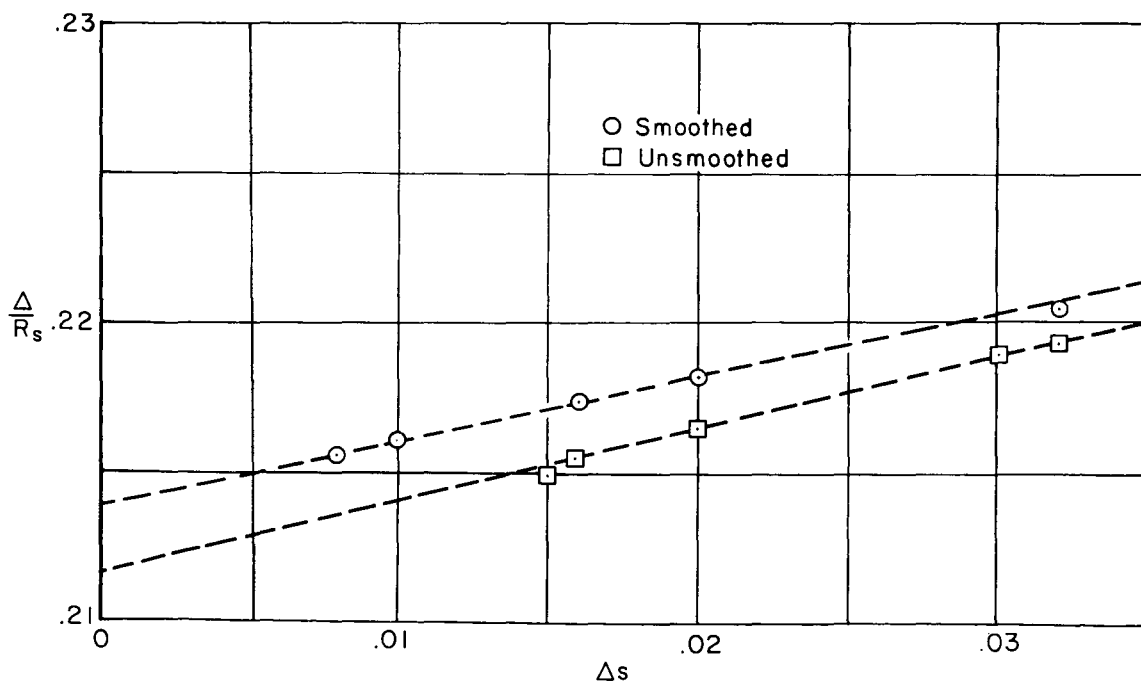
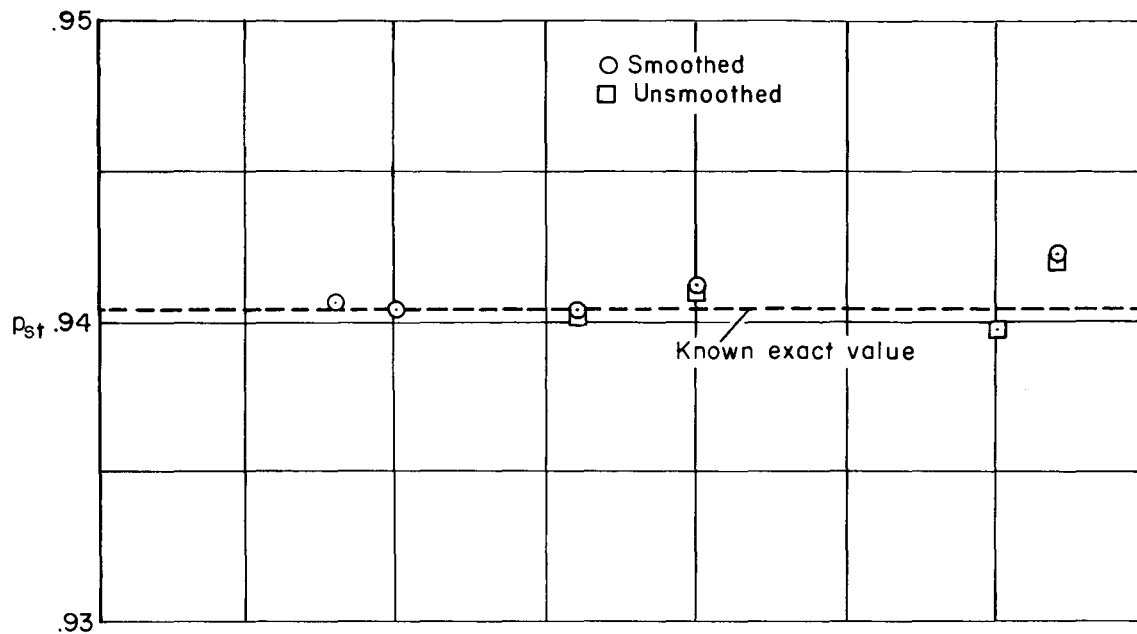
(b) $\omega(s, t)$ as function of t .

Figure 1.- Concluded.



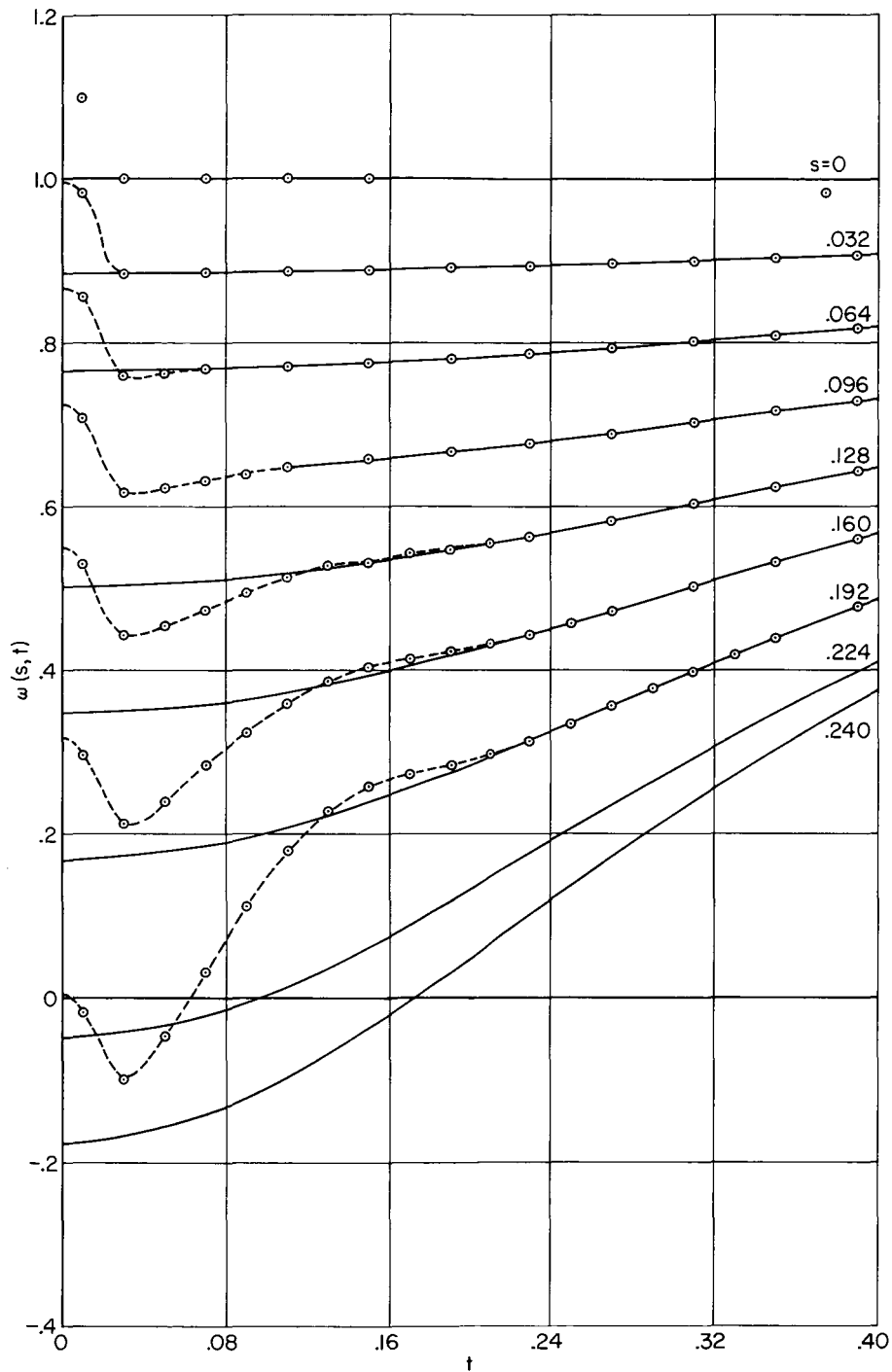
(a) Modified stream function $\omega[s, (1/2)\Delta t]$.

Figure 2.- Effect of smoothing procedure on the calculations;
 $M = 4$, $\gamma = 1.4$, $B_S = -0.06$.



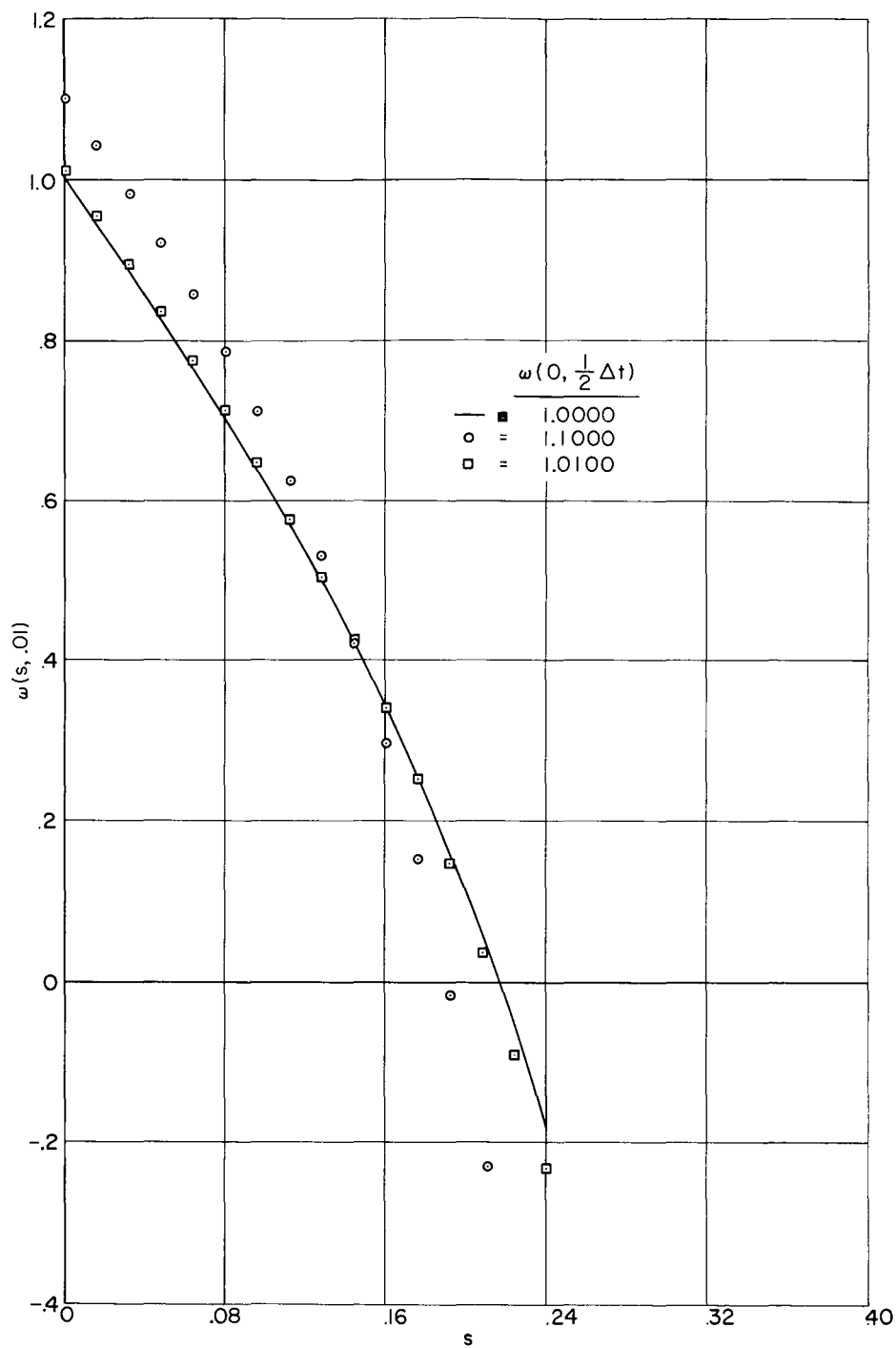
(b) Stagnation pressure p_{st} and detachment distance Δ/R_s .

Figure 2.- Concluded.



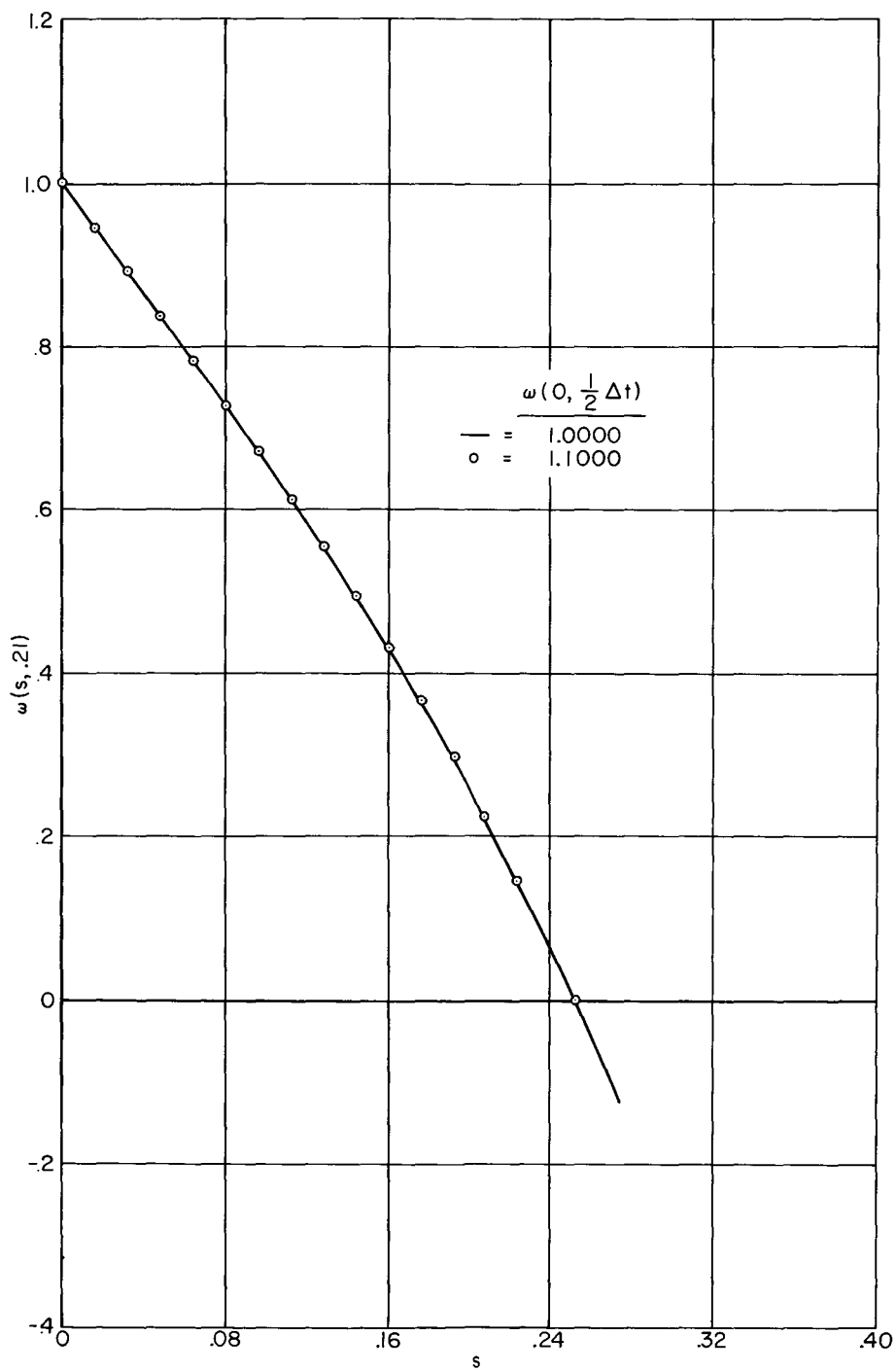
(a) $\omega(s, t)$ vs. t for $\omega[0, (1/2)\Delta t] = 1.1000$.

Figure 3.- Effect of artificial error on the smoothed calculations;
 $M = 4$, $\gamma = 1.4$, $B_S = -0.06$.



(b) $w(s, 0.01)$ vs. s for $w[0, (1/2)\Delta t] = 1.0000, 1.0100, 1.1000$.

Figure 3.- Continued.



(c) $\omega(s, 0.21)$ vs. s for $\omega[0, (1/2)\Delta t] = 1.1000$.

Figure 3.- Concluded.

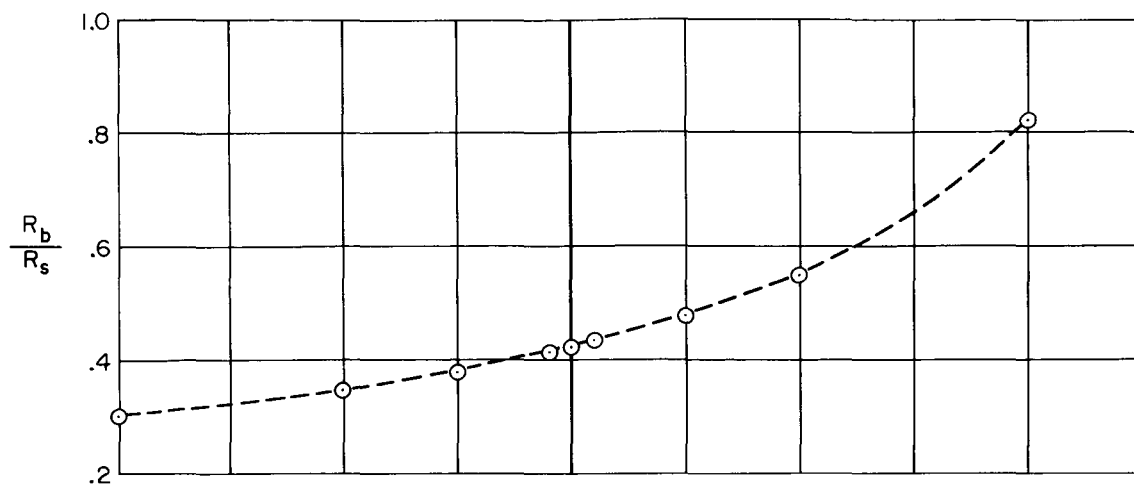
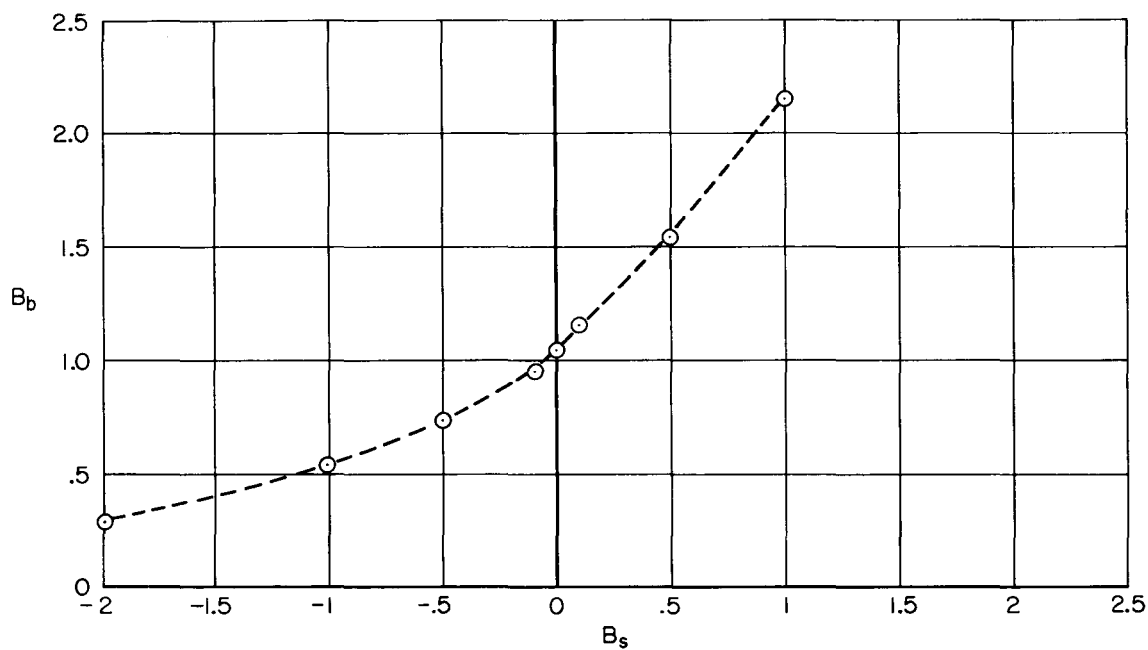
(a) Δ/R_s vs. B_s .(b) Δ/R_b vs. B_s .

Figure 4.- Effect of shock-shape parameter B_s on geometrical characteristics of bodies; $M = 4$, $\gamma = 1.4$, $A = 0$.

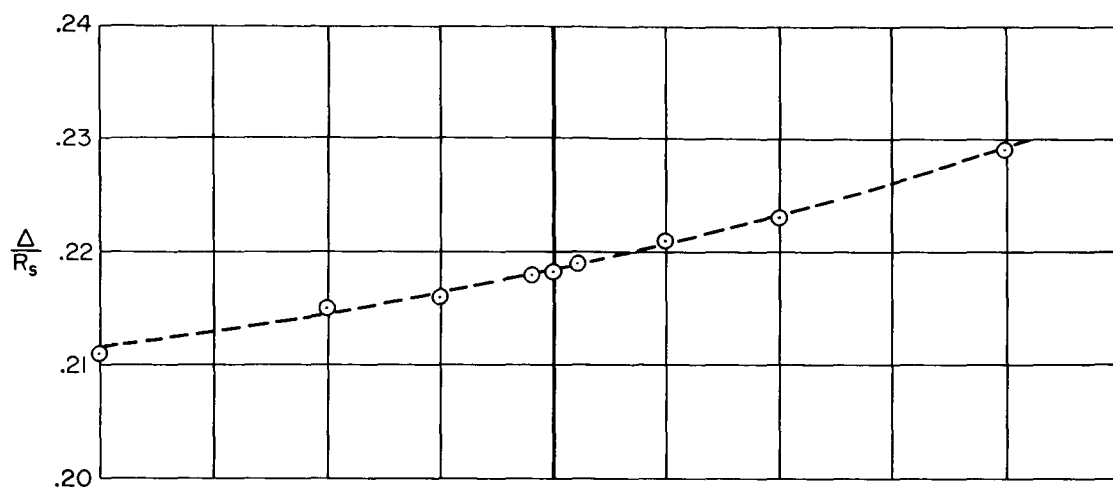
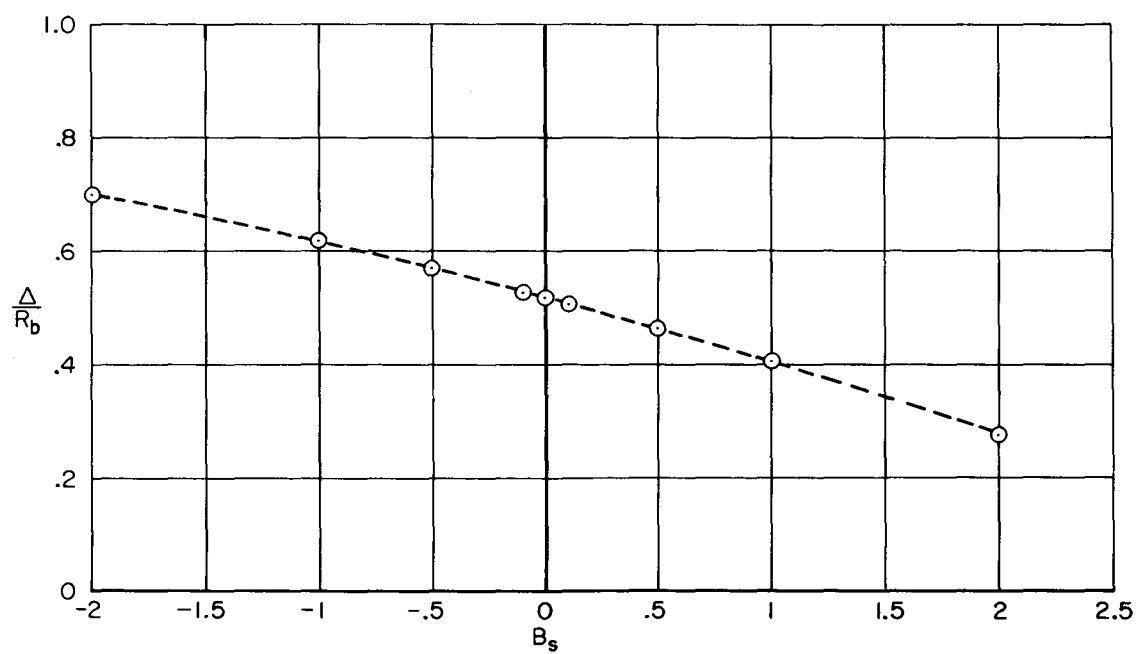
(c) R_b/R_s vs. B_s .(d) B_b vs. B_s .

Figure 4.- Concluded.

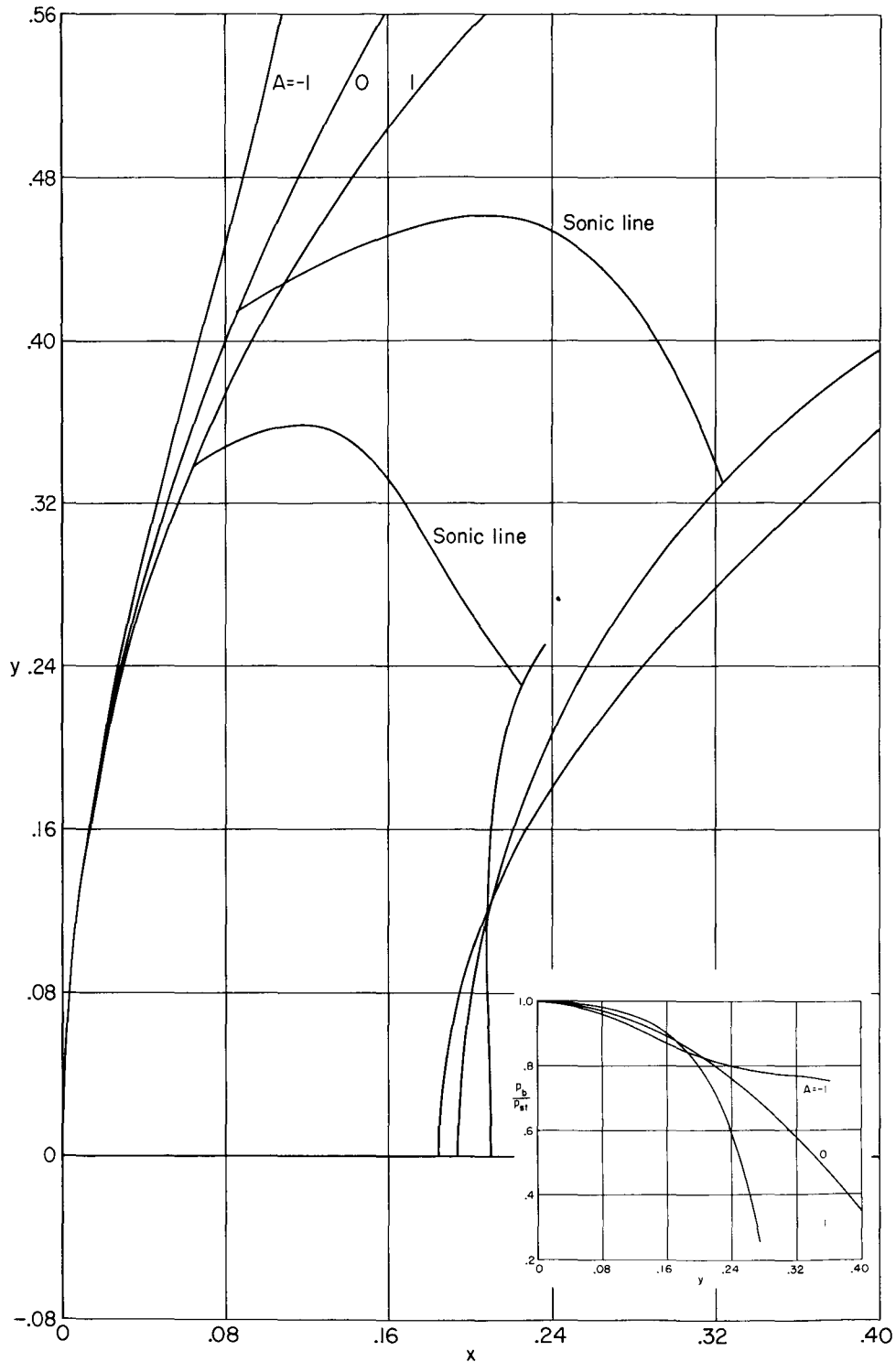


Figure 5.- Effect of parameter A on body shape; $M = 10$, $\gamma = 1.4$, $B_S = 0.1$.

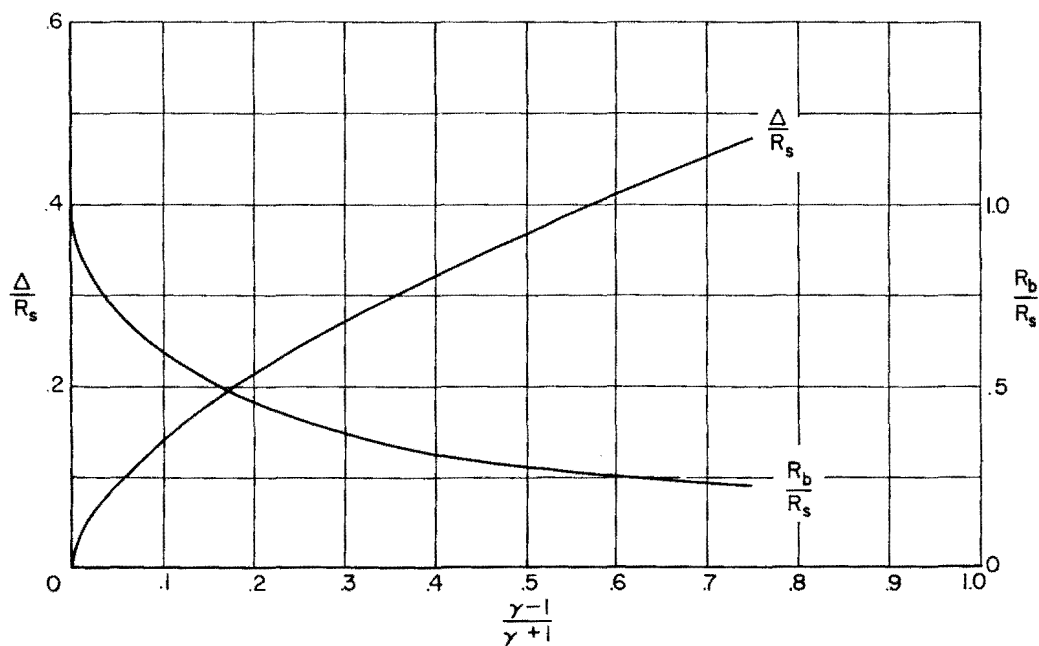
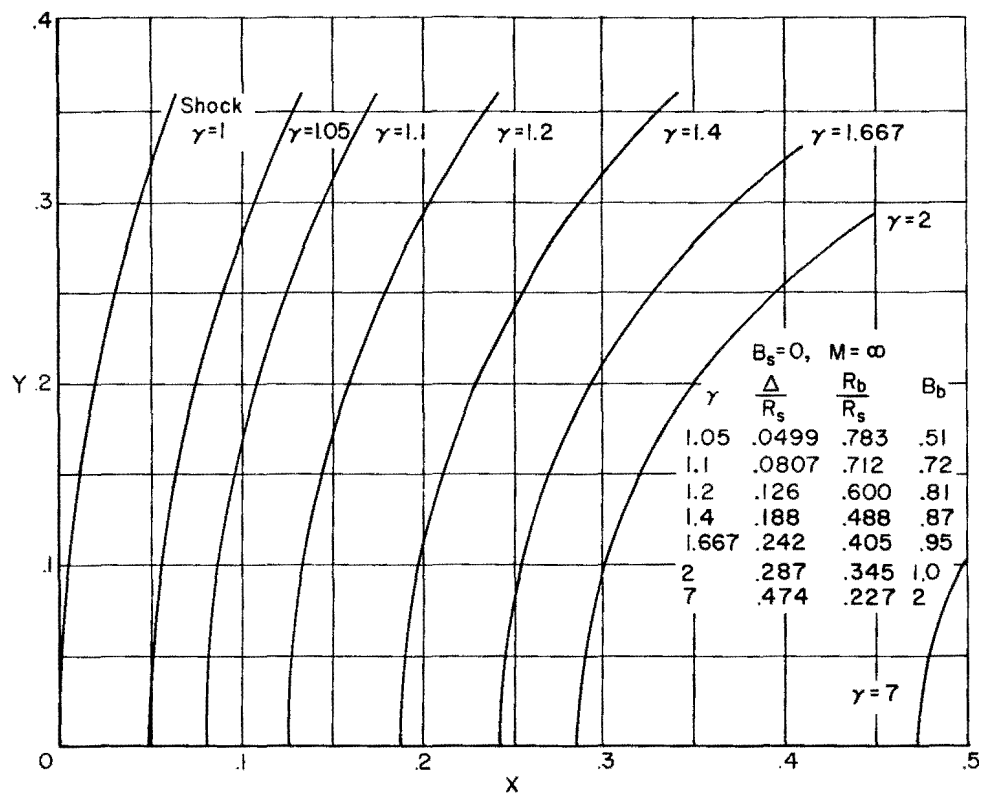


Figure 6.- Effect of variation of the adiabatic index γ on body geometry;
 $M = \infty, B_s = 0$.

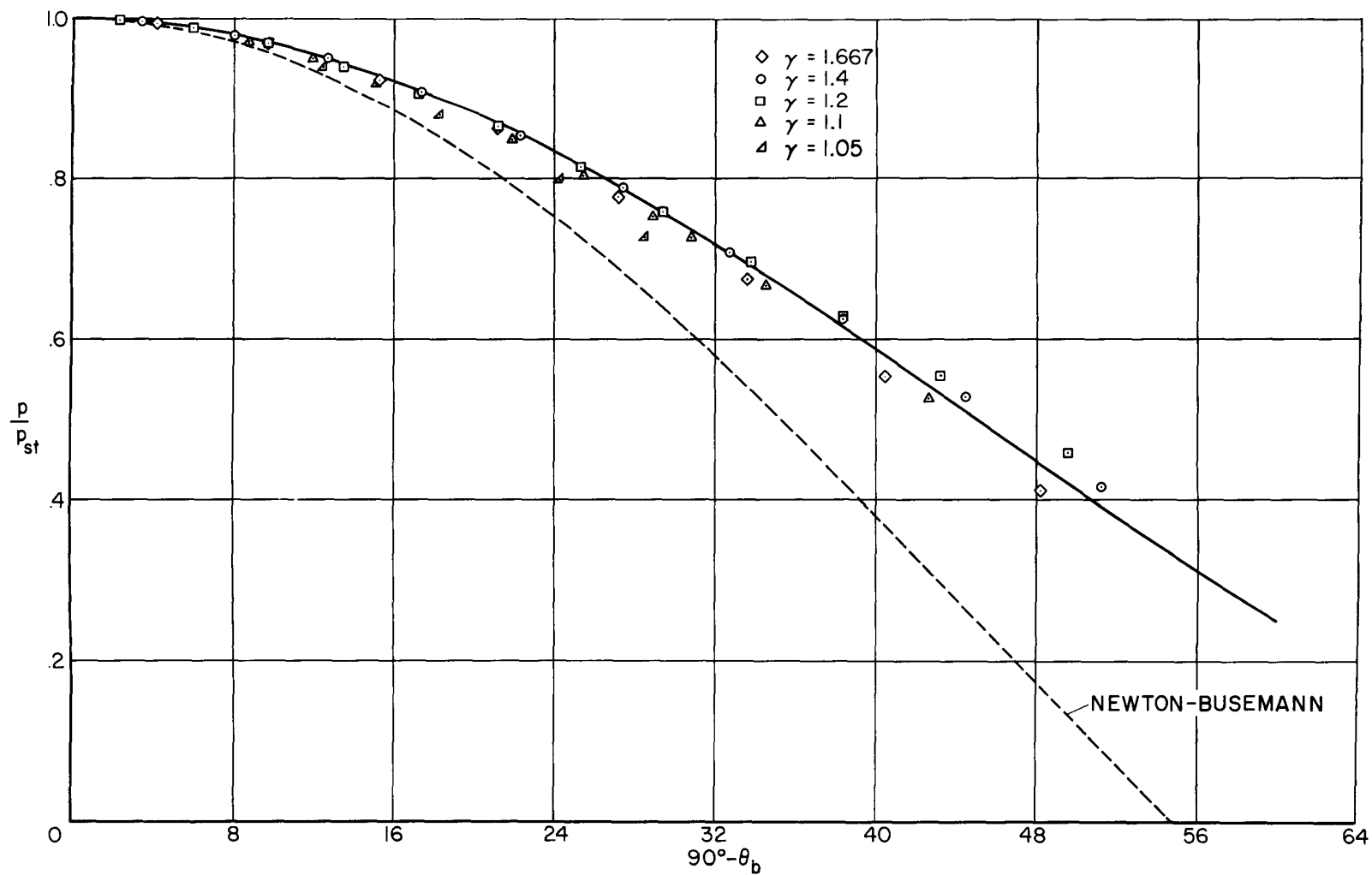


Figure 7.- Comparison of calculated pressure ratios with Newtonian approximation, circular cylinders; $M = \infty$.

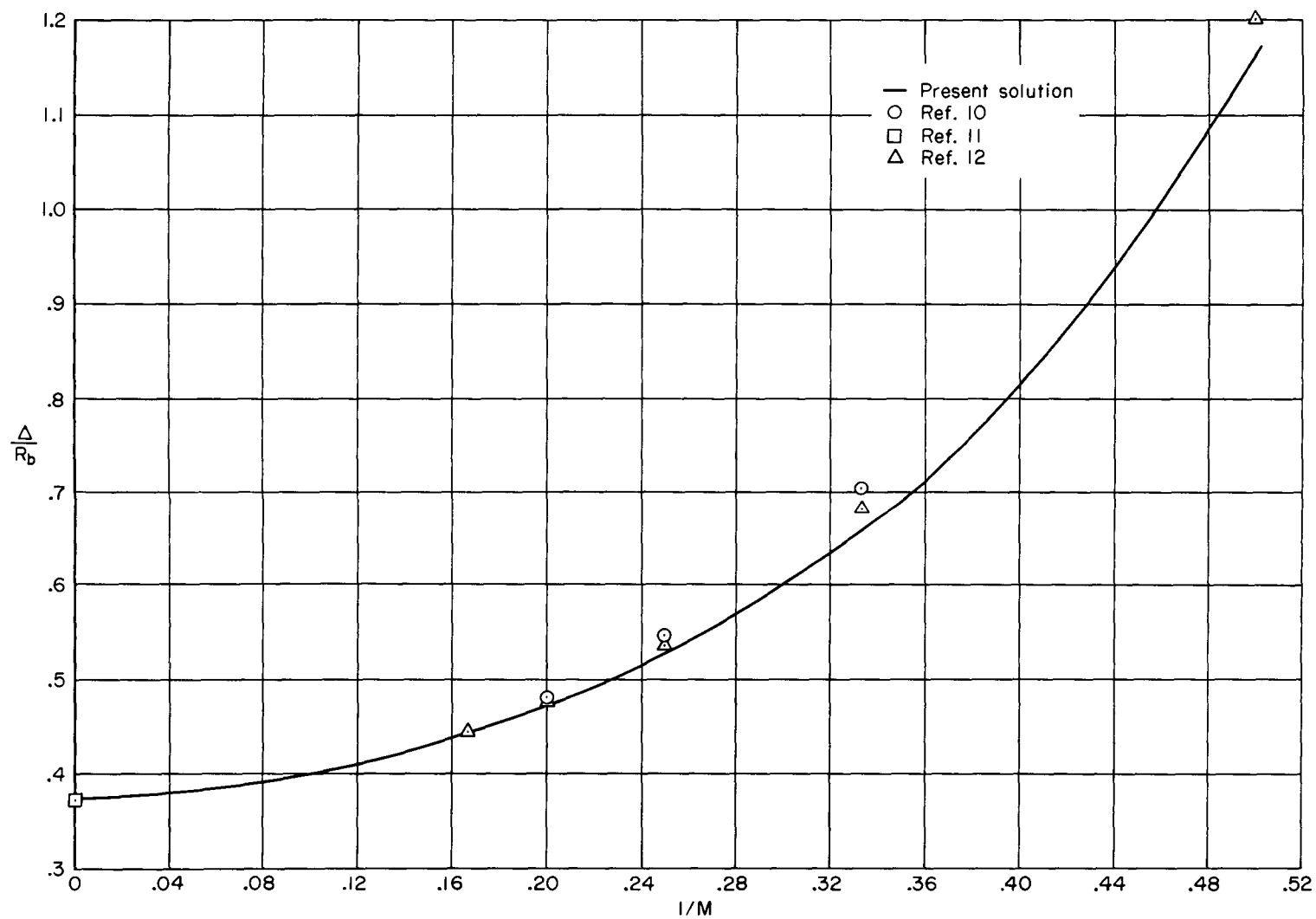


Figure 8.- Comparisons with previous solutions and experimental data for a circular cylinder.